●●●○○ Level 3 · Mathematics M3.6

Perimeter, area and volume

Find the distance round a shape, the space it covers, and the space inside a box.

Change between metric units. Find the perimeter and area of rectangles, triangles, parallelograms, trapeziums and circles, and of shapes joined from them. Find the volume of a box, change cm^3 into ml and litres, and find the paper that covers a box with no gaps.

YOU ARE READY FOR THIS BOOK IF

You can find the area of a shape on squared paper by counting squares.

One step up	M4.5 Area, volume and compound units
This book	**M3.6 Perimeter, area and volume**
One step down	M2.6 Measuring

Contents

M3.6.1 Changing metric units

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M3.6.5 Trapeziums and joined shapes

M3.6.6 Circles

M3.6.7 Volume of cuboids

M3.6.8 Surface area

Key words, A to Z

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M3.6.1 CHAPTER 1

Changing metric units

How many metres are there in 3.5 kilometres?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 35 m
- ☐ **B** 350 m
- ☐ **C** 3.5 m
- ☐ **D** 3 500 m

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Measuring, M2.6.2. Say that 1 000 m make 1 km.
- Level 3, Fractions, decimals and percentages, M3.3.5. Multiply and divide decimals by 10, 100 and 1 000.

BY THE END YOU CAN

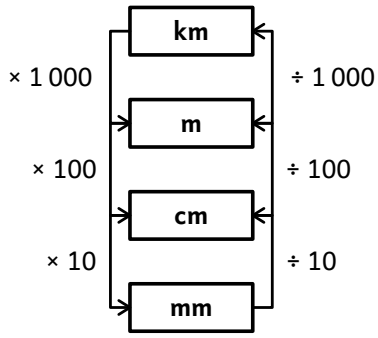
- Change 3.5 km into metres and 450 mm into centimetres
- Change between grams, kilograms and tonnes

A sign on a path says the river is 3.5 km away. How many metres is that? To find out, you **convert** the length. To convert is to change an amount from one unit to another.

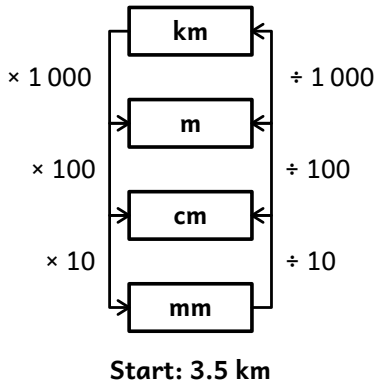
We use **metric units** for length and mass: mm, cm, m and km, and g, kg and t. Metric units go up and down in tens, hundreds and thousands. That makes them quick to convert.

DRAW IT FIRST

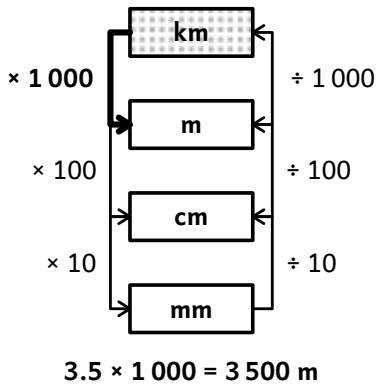
Climb down the ladder



1 Write the units in a ladder with the biggest at the top: km, m, cm, mm.



2 Start with 3.5 km. To reach metres, go down one rung.



3 Going down, you multiply by the number on the arrow: 3.5 × 1 000 = 3 500 m.

Going down the ladder, the unit gets smaller. You need more of them, so you multiply. Going up, the unit gets bigger. You need fewer of them, so you divide.

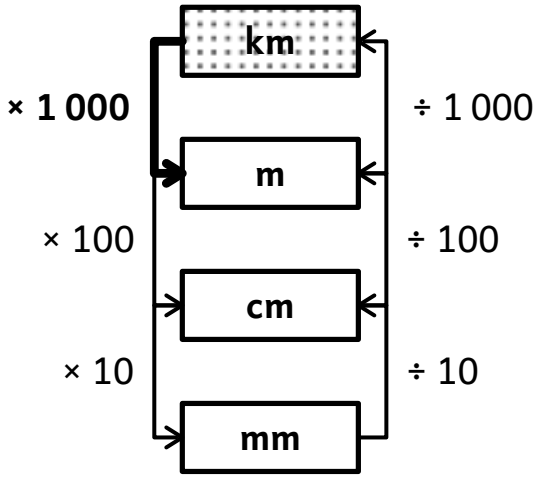
The length ladder has three steps. 1 km is 1 000 m. 1 m is 100 cm. 1 cm is 10 mm.

WORKED EXAMPLE

Change 3.5 km into metres.

1. 1 km is 1 000 m.
2. Going from km to m is going down the ladder. The unit gets smaller, so multiply.
3. 3.5 × 1 000 = 3 500 m.

Answer: 3 500 m



From km to m you go down one rung and multiply by 1 000.

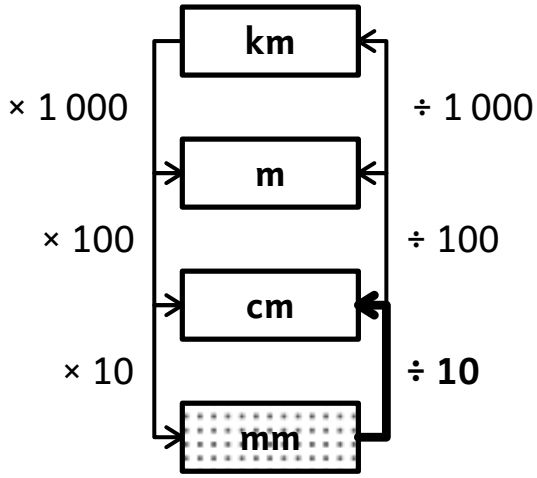
Multiplying by 1 000 moves every digit three places to the left, and zeros fill the gaps. So 3.5 becomes 3 500.

WORKED EXAMPLE

Change 450 mm into centimetres.

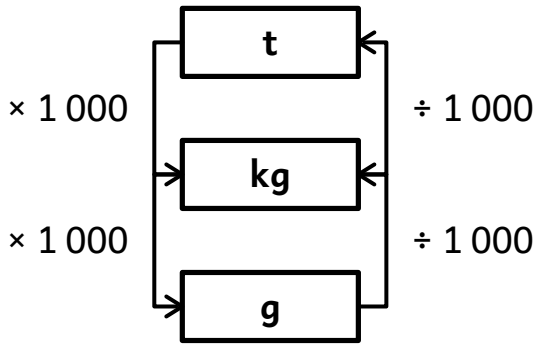
1. 10 mm is 1 cm.
2. Going from mm to cm is going up the ladder. The unit gets bigger, so divide.
3. 450 ÷ 10 = 45 cm.
4. Check by going back down: 45 × 10 = 450 mm.

Answer: 45 cm



From mm to cm you go up one rung and divide by 10.

Mass has its own ladder. 1 kg is 1 000 g. The unit for a heavy load is the **tonne** (t). One tonne is 1 000 kg, about the mass of a small car.



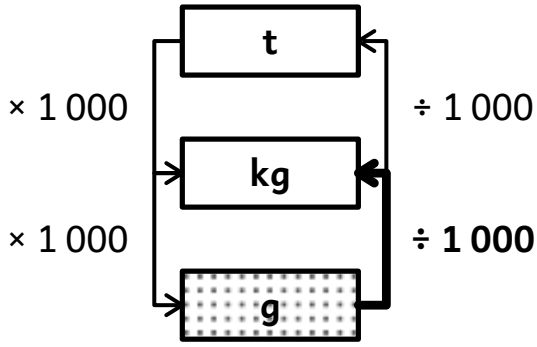
1 t is 1 000 kg. 1 kg is 1 000 g. Each step is 1 000.

WORKED EXAMPLE

Change 3 600 g into kilograms.

1. 1 kg is 1 000 g.
2. Going from g to kg is going up the ladder. The unit gets bigger, so divide.
3. 3 600 ÷ 1 000 = 3.6 kg.
4. Check by going back down: 3.6 × 1 000 = 3 600 g.

Answer: 3.6 kg



From g to kg you go up one rung and divide by 1 000.

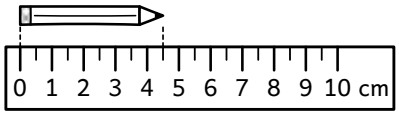
Tonnes work the same way. To change 2.5 t into kilograms, go down one rung: 2.5 × 1 000 = 2 500 kg.

SEE IT THREE WAYS

With things

Lay a ruler beside a pencil. Read its length in cm. Each cm has 10 small marks, so you can also say it in mm.

As a picture



In symbols

4.5 cm × 10 = 45 mm. And 45 mm ÷ 10 = 4.5 cm.

TRY IT

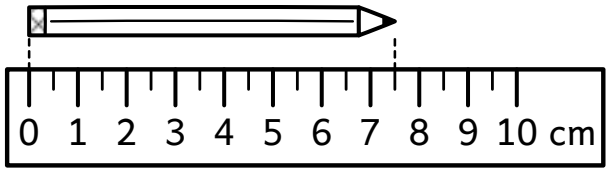
Measure in two units

You need: a ruler with cm and mm marks, a pencil, a sheet of paper.

1. Measure the length of your pencil in cm. Write it as a decimal, such as 15.2 cm.
2. Multiply by 10 to change it into mm. Write the answer.
3. Count the small marks on the ruler to read the length in mm. Compare the two answers.

What you will see: The two answers are the same length in two units. A pencil that is 7.5 cm long is also 75 mm long.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~When you change to a smaller unit, the number gets smaller.~~

Why it feels right: The word "smaller" makes people think of a smaller number.

The test that breaks it: Take a piece of string 1 m long. Measure it in cm. You count 100 cm. The unit is smaller, but you count more of them.

What is true: A smaller unit fits more times into the same length. So the number gets bigger. 1 m is 100 cm.

KEY WORDS

convert
to change an amount from one unit to another.

metric units

units that go up and down in tens, hundreds and thousands, such as m, cm and kg.

tonne
a unit of mass. 1 tonne is 1 000 kg.

BACK TO YOUR GUESS

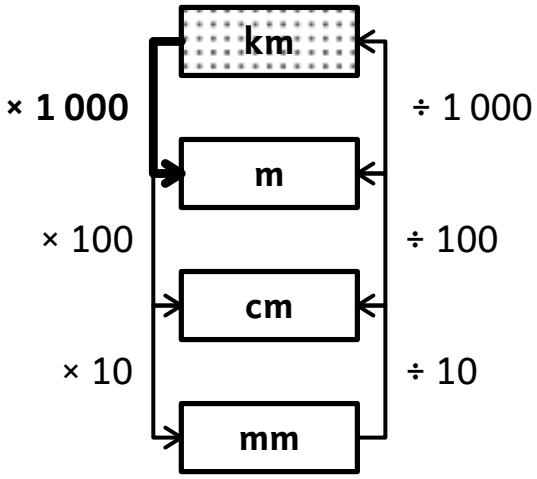
The answer is D: 3 500 m

One kilometre is 1 000 m. So 3.5 km is 3.5 × 1 000 = 3 500 m. A metre is a much smaller unit than a kilometre, so you need many more of them.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A path is 4.2 km long. How long is it in metres?



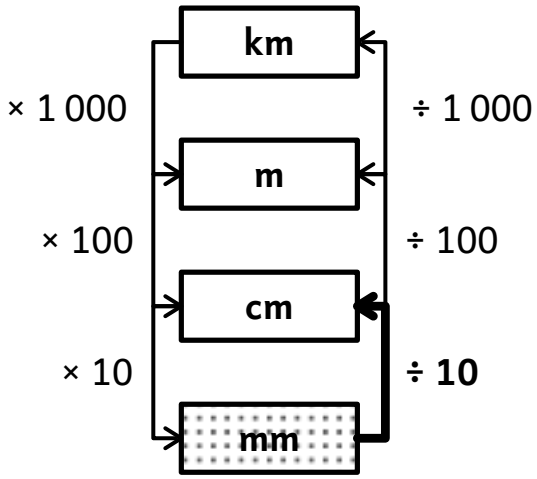
A 42 m

B 420 m

C 4 200 m

Answer at the back of the book.

2. A rope is 650 mm long. How long is it in centimetres?



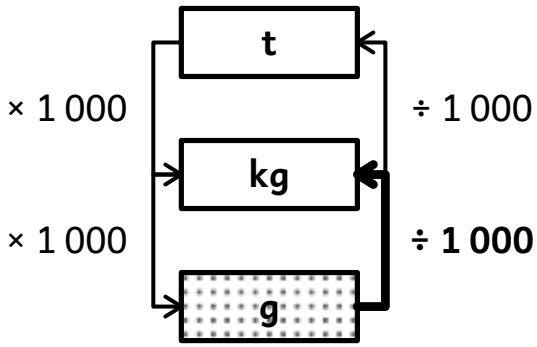
A 65 cm

B 6.5 cm

C 6 500 cm

Answer at the back of the book.

3. A bag of rice has a mass of 2 400 g. What is its mass in kilograms?



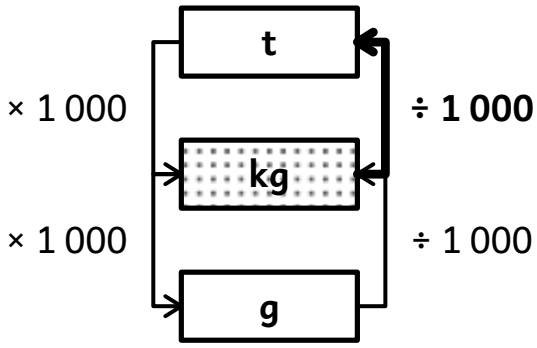
A 240 kg

B 2.4 kg

C 2 400 000 kg

Answer at the back of the book.

4. A lorry carries 4 500 kg of sand. How many tonnes is that?



A 45 t

B 450 t

C 4.5 t

Answer at the back of the book.

5. Which of these lengths is the longest?

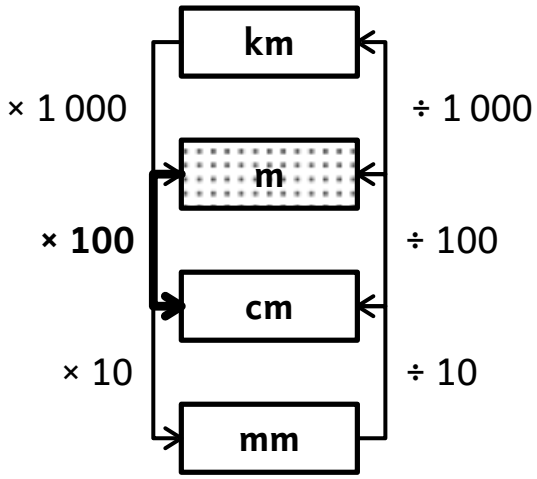
A 1 500 m

B 1.2 km

C 140 000 cm

Answer at the back of the book.

6. You change 5 m into centimetres. What happens to the number?



A It gets smaller, because a cm is a small unit.

B It gets bigger, because a cm is smaller than a m.

C It stays the same, because both are lengths.

Answer at the back of the book.

7. A girl is 1 m 62 cm tall. How tall is she in centimetres?

Answer at the back of the book.

8. A wall is 2.8 km long. How long is it in metres?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Down the ladder: multiply. Up the ladder: divide. 1 km = 1 000 m. 1 m = 100 cm. 1 cm = 10 mm. 1 t = 1 000 kg. 1 kg = 1 000 g.

Draw: The length ladder: km, m, cm, mm. Write × 1 000, × 100 and × 10 on the arrows going down. Write ÷ on the arrows going up.

M3.6.2 CHAPTER 2

Perimeter

How much fence do I need to go all the way round my garden?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Add the lengths of all the sides.
- ☐ **B** Add the length and the width.
- ☐ **C** Multiply the length by the width.
- ☐ **D** Count the squares inside the garden.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Measuring, M2.6.8. Find the perimeter of a shape by adding its sides.

BY THE END YOU CAN

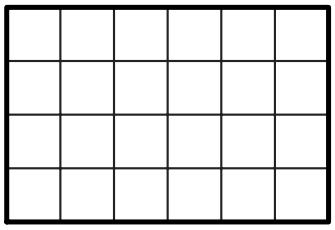
- Find the perimeter of a shape with straight sides
- Find a missing side when you know the perimeter

A farmer wants a fence round a field. She needs to know how long the fence must be. The distance all the way round a shape is its **perimeter**.

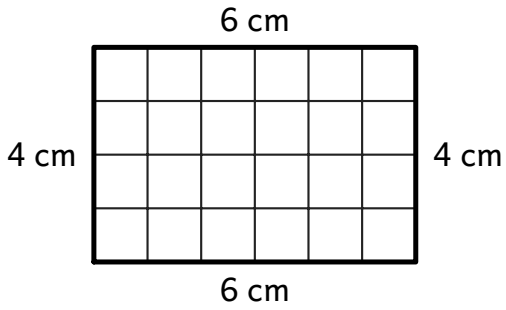
To find a perimeter, add the lengths of all the sides. Every length needs its unit, such as cm or m.

DRAW IT FIRST

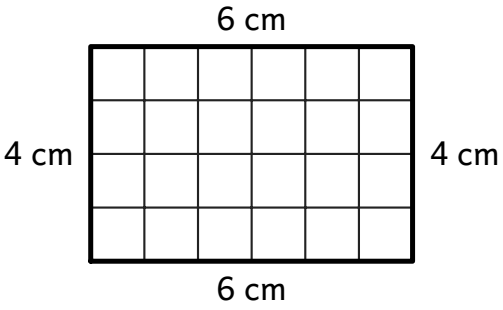
Add every side



1 Draw the shape. This rectangle is 6 cm long and 4 cm wide.



2 Write the length of every side beside it, with its unit.



6 + 4 + 6 + 4 = 20 cm

3 Add all four sides: 6 + 4 + 6 + 4 = 20 cm.

A rectangle has two long sides and two short sides. The opposite sides are equal. So the perimeter is 2 × length + 2 × width. You can also add the length and width, then double it: 2 × (length + width).

WORKED EXAMPLE

A garden is a rectangle 8 m long and 5 m wide. How much fence goes round it?

1. The fence goes round all four sides. We need the perimeter.
2. The garden has two sides of 8 m and two sides of 5 m.
3. 8 + 5 + 8 + 5 = 26 m.

Answer: 26 m



The garden. The fence is 8 + 5 + 8 + 5 = 26 m long.

A square has four sides that are all equal. So its perimeter is 4 × the side. A square with sides of 7 m has a perimeter of 4 × 7 = 28 m.

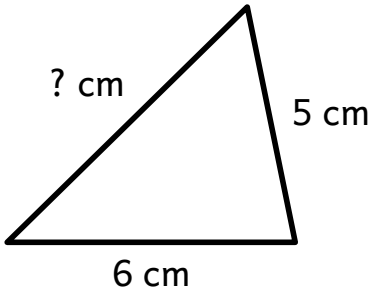
Sometimes a side is missing, and you know the perimeter instead. Add the sides you know. Then take that total away from the perimeter.

WORKED EXAMPLE

A triangle has sides of 5 cm and 6 cm, and a third side. Its perimeter is 18 cm. How long is the third side?

1. The three sides add to the perimeter: 5 + 6 + the third side = 18.
2. The two known sides add to 5 + 6 = 11 cm.
3. The third side is 18 − 11 = 7 cm.
4. Check: 5 + 6 + 7 = 18 cm.

Answer: 7 cm



perimeter 18 cm

The side marked ? cm is 18 − 5 − 6 = 7 cm.

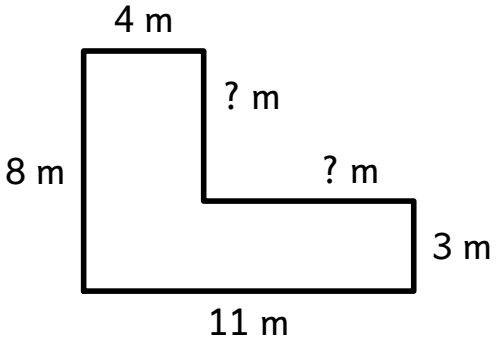
Some shapes hide a side. Look at an L shape. Two of its sides have no label. You can find each one from the sides on the opposite edge.

WORKED EXAMPLE

Find the perimeter of this L shape. Two sides are marked ? m.

1. Find the two missing sides first.
2. The flat missing side is 11 − 4 = 7 m.
3. The upright missing side is 8 − 3 = 5 m.
4. Add all six sides: 11 + 3 + 7 + 5 + 4 + 8 = 38 m.
5. A rectangle 11 m by 8 m has 2 × (11 + 8) = 38 m. The same.

Answer: 38 m



The flat ? side is 11 − 4 = 7 m. The upright ? side is 8 − 3 = 5 m.

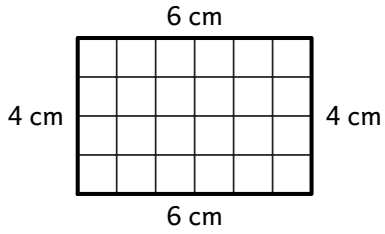
Look again at the L shape. Its perimeter is the same as the rectangle that would hold it. That works for any L shape with one corner cut out.

SEE IT THREE WAYS

With things

Wrap a string round a book cover. Pull the string straight and measure it with a ruler.

As a picture



In symbols

Perimeter = 6 + 4 + 6 + 4 = 20 cm. Or 2 × (6 + 4) = 20 cm.

TRY IT

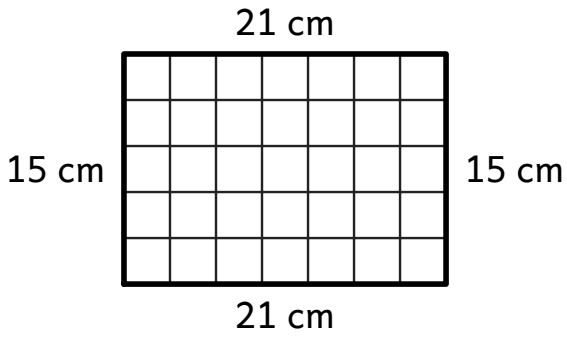
The string round a book

You need: a book or an exercise book, a piece of string, a ruler, a pencil.

1. Measure the long side and the short side of the cover in cm.
2. Add the four sides to find the perimeter.
3. Lay the string round the edge of the cover. Cut it where it meets.
4. Straighten the string and measure it with the ruler.

What you will see: The string and the sum give the same length. A cover is 21 cm by 15 cm. Its perimeter is 21 + 15 + 21 + 15 = 72 cm.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~The perimeter of a rectangle is its length plus its width.~~

Why it feels right: Those are the two numbers you see on the picture.

The test that breaks it: Walk round a rectangle that is 5 m by 3 m. You walk 5 m, then 3 m, then 5 m, then 3 m. That is 16 m, not 8 m.

What is true: A rectangle has four sides. Add all four, or add the length and width and then double the total.

KEY WORDS

perimeter
the distance all the way round a shape.

rectangle
a four sided shape with four right angles. Its opposite sides are equal.

missing side
a side with no length written. You work it out from the sides you know.

BACK TO YOUR GUESS

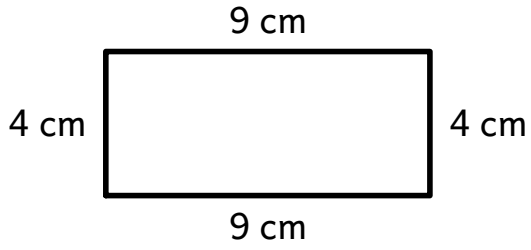
The answer is A: Add the lengths of all the sides.

A fence goes round every side. So you add the lengths of all the sides. This total is the perimeter. Multiplying or counting squares gives the space inside, which is a different job.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. What is the perimeter of this rectangle?



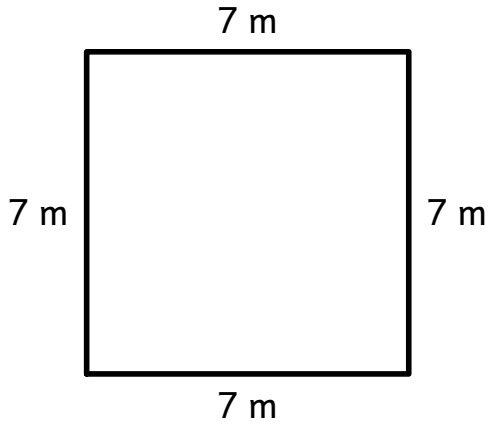
A 13 cm

B 26 cm

C 36 cm

Answer at the back of the book.

2. A square has sides of 7 m. What is its perimeter?



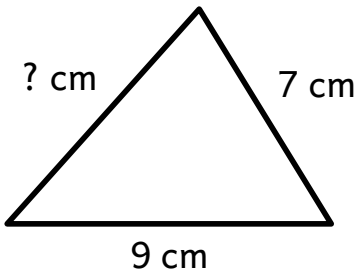
A 14 m

B 49 m

C 28 m

Answer at the back of the book.

3. A triangle has sides of 9 cm and 7 cm and a third side. Its perimeter is 24 cm. How long is the third side?



perimeter 24 cm

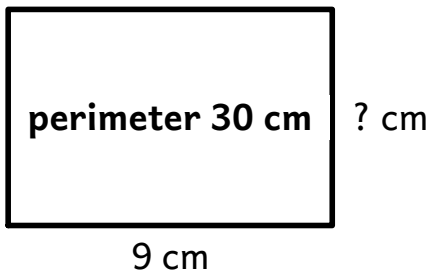
A 8 cm

B 16 cm

C 40 cm

Answer at the back of the book.

4. A rectangle has a perimeter of 30 cm. Its length is 9 cm. How wide is it?



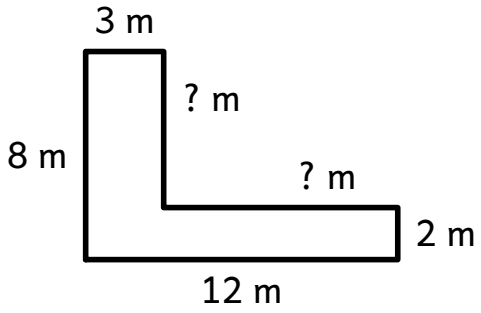
A 21 cm

B 6 cm

C 12 cm

Answer at the back of the book.

5. What is the perimeter of this L shape? Two sides are marked ? m.



A 25 m

B 96 m

C 40 m

Answer at the back of the book.

6. Which job needs the perimeter?

A Putting a fence round a field

B Covering a floor with tiles

C Filling a box with sand

Answer at the back of the book.

7. A rectangle is 12 m long and 7 m wide. What is its perimeter?

Answer at the back of the book.

8. **From an earlier chapter**

A rectangle is 1.5 m long and 80 cm wide. What is its perimeter in centimetres?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Perimeter is the distance all the way round. Add every side and write the unit. Rectangle: 2 × (length + width). Square: 4 × side. Missing side: perimeter minus the sides you know.

Draw: A rectangle 6 cm by 4 cm with each side labelled. Write 6 + 4 + 6 + 4 = 20 cm under it.

M3.6.3 CHAPTER 3

Area of rectangles

How many tiles will cover my floor?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Add the length and the width of the floor.
- ☐ **B** Add all four sides of the floor.
- ☐ **C** Multiply the length by the width of the floor.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Measuring, M2.6.8. Find the area of a shape by counting squares.
- Level 3, Long multiplication and division, M3.2.1. Multiply multiples of ten, like 30 × 40.

BY THE END YOU CAN

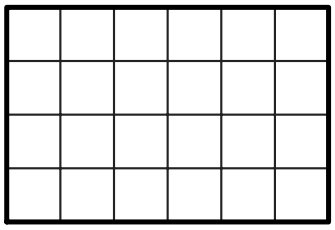
- Find the area of a rectangle as length × width
- Find the area of an L shape by splitting it
- Explain why 1 m^2 is 10 000 cm^2

You want to cover a floor with square tiles. How many tiles do you need? The space inside a shape is its ***area***. We measure area by counting squares.

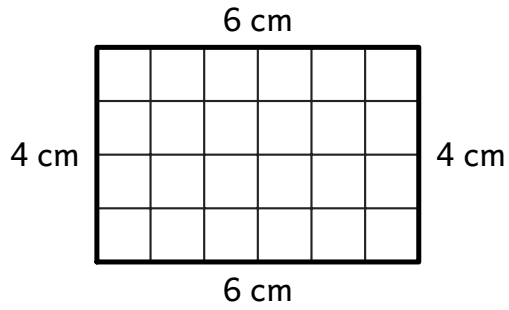
A **square centimetre** is a square that is 1 cm long and 1 cm wide. We write it as 1 cm^2. An area of 24 cm^2 means 24 of these squares fit inside the shape.

DRAW IT FIRST

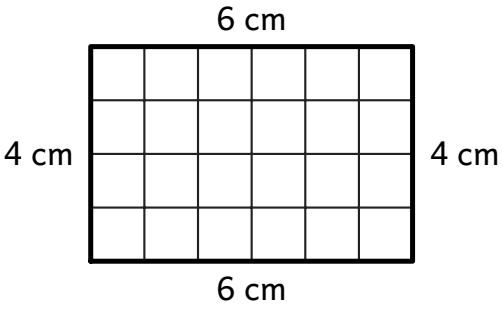
Count the squares in rows



1 Draw the rectangle on squared paper. Each square is 1 cm by 1 cm.



2 One row has 6 squares. There are 4 rows.



6 × 4 = 24 cm²

3 Count all the squares with a sum: 6 × 4 = 24 cm².

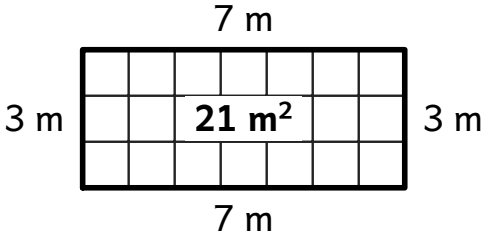
Rows of equal size make a short cut. The number of squares in a row is the length. The number of rows is the width. So the area of a rectangle is its length × its width.

WORKED EXAMPLE

A floor is a rectangle 7 m long and 3 m wide. What is its area?

1. Imagine squares that are 1 m by 1 m. Each is a **square metre**, written 1 m^2.
2. There are 3 rows with 7 squares in each row.
3. $7 \times 3 = 21$ m^2.

Answer: 21 m^2



Each square is 1 m by 1 m. There are 3 rows of 7 squares.

A square is a rectangle with equal sides. A square with sides of 6 m has an area of $6 \times 6 = 36$ m^2. Always write the unit with a small 2, such as cm^2 or m^2, because you are counting squares.

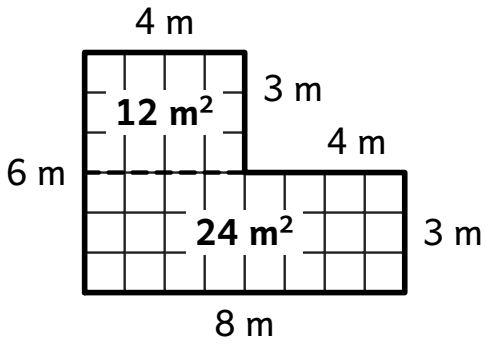
Some shapes are not rectangles. An L shape is made of two rectangles joined together. To find its area, split it into rectangles, find each area, and add them.

WORKED EXAMPLE

Find the area of this L shape by splitting it with a flat line.

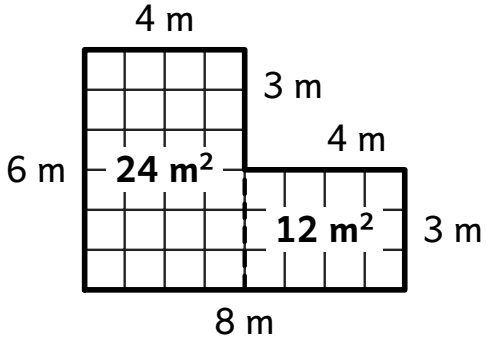
1. Draw a dashed line to split the L into two rectangles.
2. The bottom rectangle is 8 m by 3 m: $8 \times 3 = 24$ m^2.
3. The top rectangle is 4 m by 3 m: $4 \times 3 = 12$ m^2.
4. Add the two areas: $24 + 12 = 36$ m^2.

Answer: 36 m^2



Split with a flat line: 24 + 12 = 36 m².

You can split the L another way, with an upright line. The two parts change, but the total does not.



Split with an upright line: 4 × 6 = 24 and 4 × 3 = 12. The total is 36 m² again.

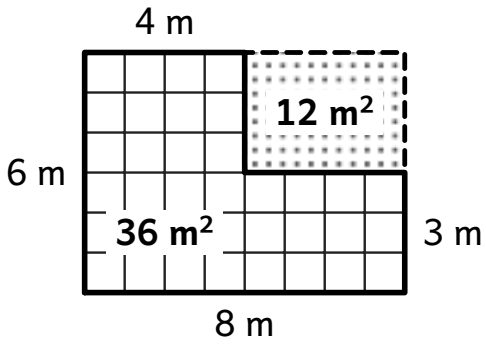
There is a third way. Fill in the missing corner to make one big rectangle. Find its area, then take the corner away.

WORKED EXAMPLE

Find the area of the same L shape by filling in the corner.

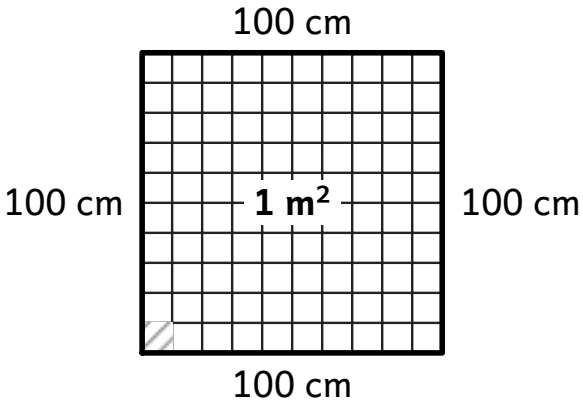
1. Draw the rectangle that would hold the L. It is 8 m by 6 m.
2. Its area is 8 × 6 = 48 m².
3. The corner that is missing is 4 m by 3 m: 4 × 3 = 12 m².
4. Take the corner away: 48 − 12 = 36 m².

Answer: 36 m²

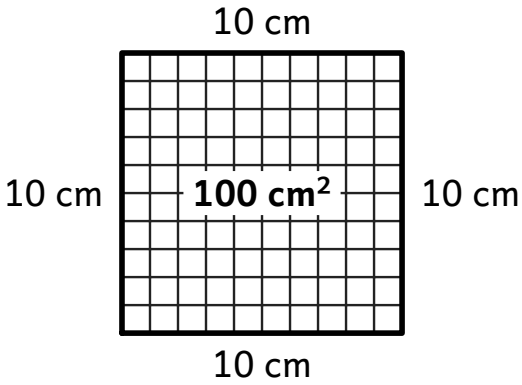


The whole rectangle is 8 × 6 = 48 m^2. The dotted corner is 4 m by 3 m, so 4 × 3 = 12 m^2. The L is 48 − 12 = 36 m^2.

Now think small. A square metre is *a* square that is 1 m long and 1 m wide. But 1 m is also 100 cm. So *a* square metre is *a* square 100 cm long and 100 cm wide.



The square metre. Each small square is 10 cm by 10 cm.



One small square from the picture above, made bigger. It holds 10 × 10 = 100 squares of 1 cm^2.

WORKED EXAMPLE

Why is 1 m^2 the same as 10 000 cm^2?

1. 1 m is 100 cm.
2. A square metre is 100 cm long and 100 cm wide.
3. Cut it into squares of 10 cm by 10 cm. You get 10 across and 10 down, which is 100 squares.
4. Each small square is 10 × 10 = 100 cm^2.
5. 100 squares of 100 cm^2 each make 100 × 100 = 10 000 cm^2.

Answer: 10 000 cm^2

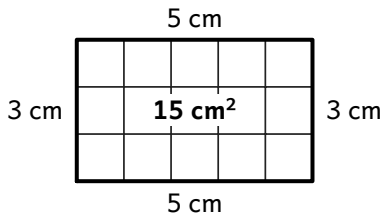
So 1 m^2 is not 100 cm^2. It is 100 times 100, which is 10 000 cm^2. Area counts squares. A square has a length and a width, so the 100 is used twice.

SEE IT THREE WAYS

With things

On squared paper, make 3 rows with 5 squares in each row. Count all the squares.

As a picture



In symbols

Area = length × width = 5 × 3 = 15 cm^2.

TRY IT

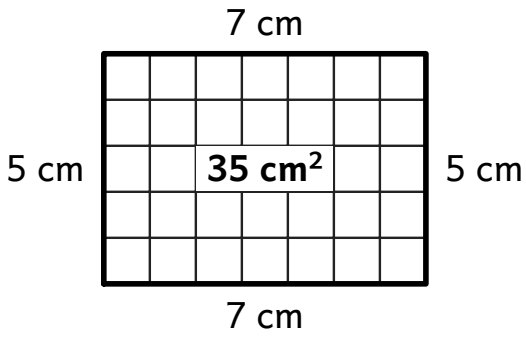
Count the squares, then multiply

You need: squared paper with squares 1 cm by 1 cm, a pencil, a ruler.

1. Draw a rectangle that is 7 squares long and 5 squares wide.
2. Count the squares one by one. Write the number.
3. Now multiply 7 × 5. Compare the two answers.

What you will see: Counting and multiplying give the same area, 35 cm^2. Multiplying is faster, and it works for any size.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~A shape with a longer perimeter must have a bigger area.~~

Why it feels right: A longer edge seems to hold more.

The test that breaks it: Draw a strip 8 cm by 1 cm and a square 3 cm by 3 cm. The strip has the longer perimeter, 18 cm against 12 cm. The square has the bigger area, 9 cm^2 against 8 cm^2.

What is true: Perimeter is the distance round. Area is the space inside. They are two different measures, and one does not tell you the other.

KEY WORDS

area
the space inside a shape, counted in squares.

square centimetre
a square 1 cm long *and* 1 cm wide. Written cm^2.

square metre
a square 1 m long *and* 1 m wide. Written m^2.

BACK TO YOUR GUESS

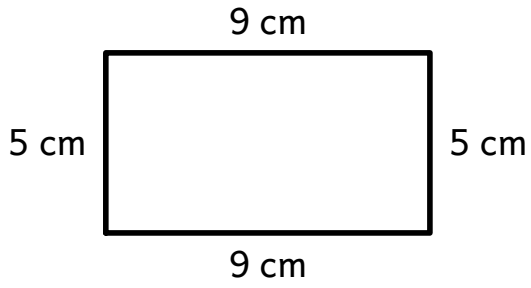
The *answer* is C: Multiply the length by the width of the floor.

Tiles cover the space inside the floor. A floor 7 tiles long *and* 3 tiles wide *has* 3 rows of 7 tiles. That is 7 × 3 = 21 tiles. So you multiply the length by the width.

CHECK YOURSELF

Try *each* one before you look. A wrong *answer* tells you where to go next.

1. What is the *area* of this rectangle?



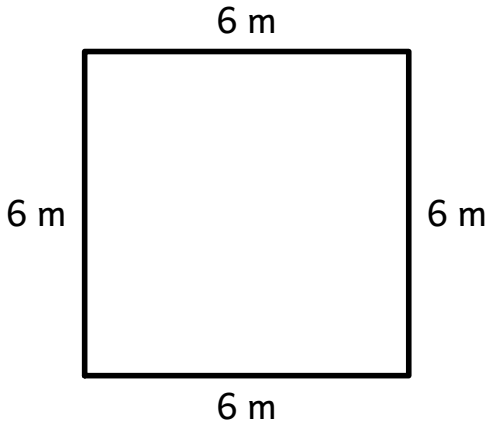
A 45 cm^2

B 14 cm^2

C 28 cm^2

Answer *at* the *back* of the book.

2. A square has sides of 6 m. What is its area?



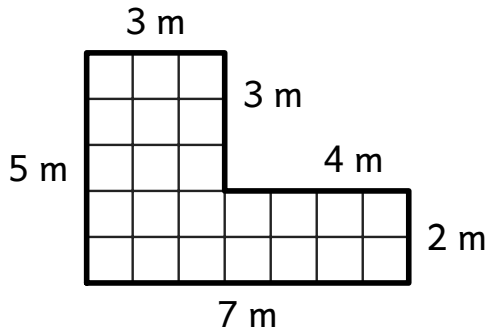
A 12 m^2

B 36 m^2

C 24 m^2

Answer at the back of the book.

3. What is the area of this L shape?



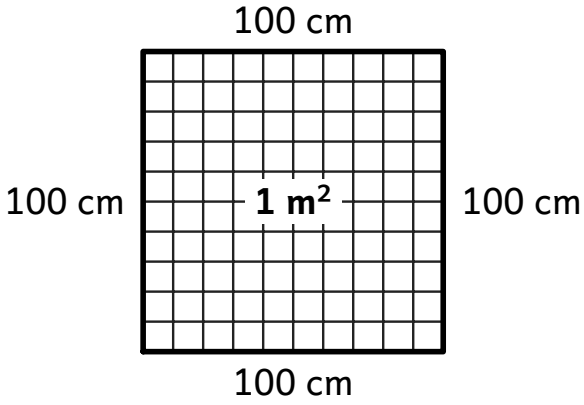
A 23 m^2

B 35 m^2

C 24 m^2

Answer at the back of the book.

4. How many square centimetres are in one square metre?



A 100 cm^2

B 1 000 cm^2

C 10 000 cm^2

Answer at the back of the book.

5. What is one square centimetre?

A A line one centimetre long

B A square with sides one centimetre long

C A cube with edges one centimetre long

Answer at the back of the book.

6. A rectangle has an area of 40 cm^2. Its length is 8 cm. How wide is it?

A 32 cm

B 320 cm

C 5 cm

Answer at the back of the book.

7. A rectangle is 9 cm long and 4 cm wide. What is its area?

Answer at the back of the book.

8. **From an earlier chapter**

A rectangle is 12 m long and 5 m wide. What is its perimeter?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Area is the space inside, counted in squares. Rectangle: length × width. L shape: split it, or fill and take away. 1 m^2 = 100 cm × 100 cm = 10 000 cm^2.

Draw: A rectangle on squared paper, 6 by 4, with its squares drawn. Write 6 × 4 = 24 cm^2 beside it.

M3.6.4 CHAPTER 4

Area of triangles and parallelograms

A leaning shape has the same base and height as a rectangle. Does it hold less space?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** No. It covers the same space, because the base and the height are the same.
- ☐ **B** Yes. It is leaning, so it covers less.
- ☐ **C** Yes. The slanted sides make it smaller.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Shapes and angles, M2.5.5. Name a square, rectangle, rhombus, parallelogram and trapezium.

BY THE END YOU CAN

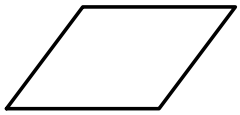
- Find the area of a parallelogram as base × height
- Find the area of a triangle as half of base × height

A **parallelogram** is a four sided shape with two pairs of parallel sides. It looks like a rectangle that leans. Does it hold less space than a rectangle with the same base and height?

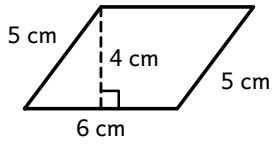
One side of the shape is its **base**. The **height** is the distance straight up from the base to the top side. It is measured at a right angle to the base. A dashed line shows it.

DRAW IT FIRST

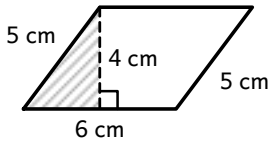
Turn *a* parallelogram into *a* rectangle



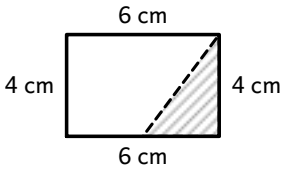
1 Draw a parallelogram. Its base is 6 cm.



2 Draw the height from the top corner straight down to the base. It is 4 cm. The slanted side is 5 cm, which is not the height.



Cut it off. Slide it across.



3 Cut off the triangle at one end and slide it to the other end. You make a rectangle 6 cm by 4 cm.

The rectangle has the same area as the parallelogram, because nothing was added or lost. The rectangle is 6 cm long and 4 cm wide. So its area is $6 \times 4 = 24$ cm^2.

Pushing a frame over is different. Its sides keep their length, so the height falls and the space inside shrinks.

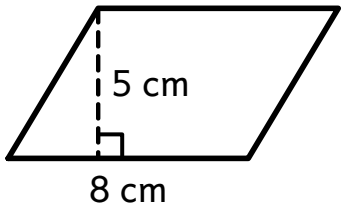
The area of a parallelogram is base × height. Use the height, not the slanted side. The slanted side is longer than the height, so it gives an answer that is too big.

WORKED EXAMPLE

A parallelogram has a base of 8 cm and a height of 5 cm. What is its area?

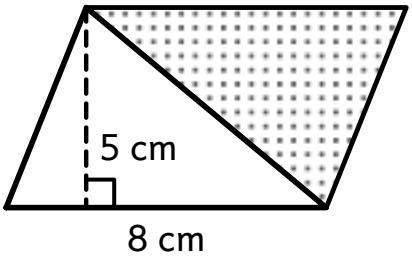
1. The base is 8 cm. The height is the dashed line, 5 cm.
2. The area is base × height.
3. $8 \times 5 = 40$ cm^2.

Answer: 40 cm^2



The height is the dashed line, at a right angle to the base.

Now a triangle. Make a copy of any triangle and turn the copy half way round. Fit the two together along a side. They make a parallelogram.



Two copies of the triangle make a parallelogram with base 8 cm and height 5 cm.

The parallelogram is $8 \times 5 = 40$ cm^2. The triangle is only half of it. So the area of a triangle is half of base × height.

WORKED EXAMPLE

A triangle has a base of 8 cm and a height of 5 cm. What is its area?

1. Two copies make a parallelogram with base 8 cm and height 5 cm.
2. That parallelogram has an area of $8 \times 5 = 40$ cm^2.
3. The triangle is half of it: $40 \div 2 = 20$ cm^2.

Answer: 20 cm^2

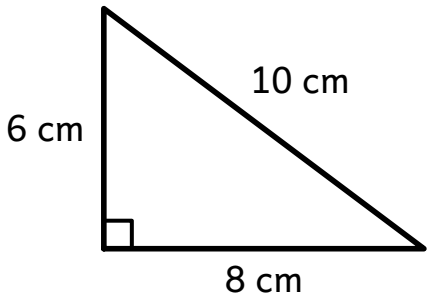
A right angled triangle has two sides at the right angle. These can be the base and the height. The third side is slanted, so you do not use it.

WORKED EXAMPLE

A right angled triangle has sides of 6 cm and 8 cm at the right angle. The slanted side is 10 cm. What is its area?

1. The two sides at the right angle are the base and the height: 8 cm and 6 cm.
2. The slanted side, 10 cm, is not used.
3. Base × height = $8 \times 6 = 48$ cm^2.
4. The triangle is half of that: $48 \div 2 = 24$ cm^2.
5. Check: a rectangle 8 cm by 6 cm has an area of 48 cm^2. The triangle is half of it, 24 cm^2.

Answer: 24 cm^2



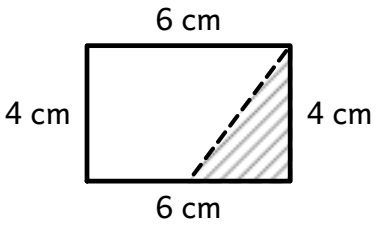
The base and the height are the two sides at the right angle.

SEE IT THREE WAYS

With things

Cut a parallelogram from paper. Cut a triangle off one end and slide it to the other end.

As a picture



In symbols

Parallelogram: area = base × height. Triangle: area = half of base × height.

TRY IT

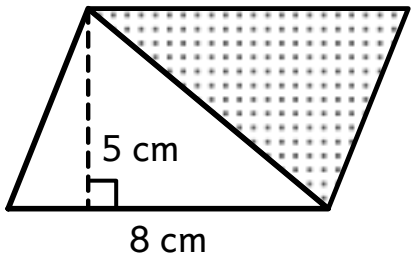
Two triangles make a parallelogram

You need: a sheet of paper, scissors, a pencil.

1. Draw a triangle of any shape on the paper. Cut it out.
2. Trace round it on a second sheet. Cut out the copy.
3. Turn the copy half way round, flat on the table. Do not flip it over. Fit one of its sides to the same side of the first triangle.
4. Look at the new shape.

What you will see: The two triangles fit together without a gap and make a parallelogram. Each triangle is half of it. This works for every triangle you try.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~To find the area of a parallelogram, multiply the base by the slanted side.~~

Why it feels right: The slanted side is a real side of the shape. It often has a length written beside it.

The test that breaks it: Cut the triangle off and slide it across. The rectangle is 6 cm by 4 cm, so the area is 24 cm^2. Using the slanted side gives 6 × 5 = 30, which is more squares than the shape holds.

What is true: Use the height, the dashed line that meets the base at a right angle.

KEY WORDS

parallelogram
a four sided shape with two pairs of parallel sides.

base
the side of a shape that you measure the height from.

height
the distance from the base straight up to the top, at a right angle to the base.

BACK TO YOUR GUESS

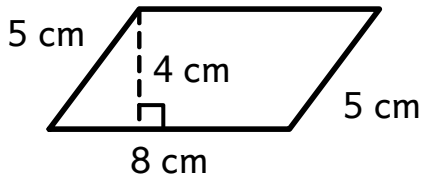
The answer is A: No. It covers the same space, because the base and the height are the same.

Cut a triangle off one end of the leaning shape and slide it to the other end. You get the rectangle. Nothing was added or lost, so the area is the same.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. What is the area of this parallelogram?



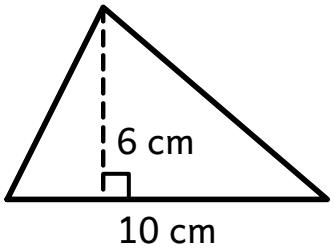
A 40 cm^2

B 26 cm^2

C 32 cm^2

Answer at the back of the book.

2. A triangle has a base of 10 cm and a height of 6 cm. What is its area?



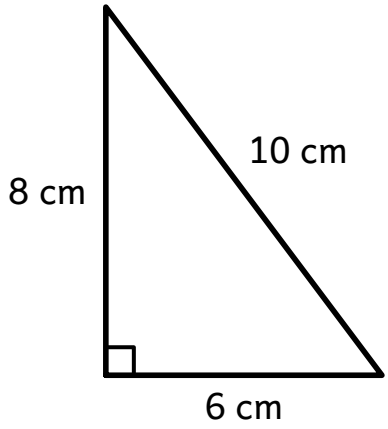
A 60 cm^2

B 30 cm^2

C 16 cm^2

Answer at the back of the book.

3. This triangle has a right angle. Its sides at the right angle are 6 cm and 8 cm. The slanted side is 10 cm. What is its area?



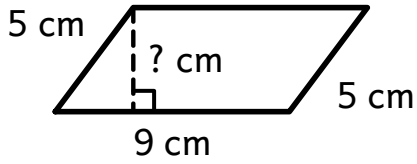
A 40 cm^2

B 48 cm^2

C 24 cm^2

Answer at the back of the book.

4. A parallelogram has a base of 9 cm and an area of 36 cm^2. How long is its height?



A 4 cm

B 5 cm

C 27 cm

Answer at the back of the book.

5. A rectangle is 6 cm by 4 cm. A parallelogram has a base of 6 cm and a height of 4 cm. Which has the larger area?

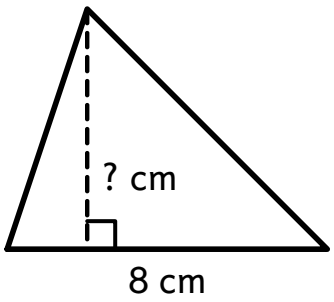
A The rectangle has the larger area.

B They have the same area.

C The parallelogram has the larger area.

Answer at the back of the book.

6. A triangle has an area of 24 cm^2 and a base of 8 cm. How long is its height?



A 6 cm

B 3 cm

C 12 cm

Answer at the back of the book.

7. A triangle has a base of 10 cm and a height of 4 cm. What is its area?

Answer at the back of the book.

8. **From an earlier chapter**

A rectangle is 11 cm long and 6 cm wide. What is its area?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Height is at a right angle to the base, never the slanted side. Parallelogram: base × height. Triangle: half of base × height.

Draw: A parallelogram with a dashed height and a small square at the right angle. Beside it, a triangle with its height.

M3.6.5 CHAPTER 5

Trapeziums and joined shapes

How do I find the area of a flower bed with two parallel sides of different lengths?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Multiply the longer parallel side by the height.
- ☐ **B** Add the two parallel sides, multiply by the height, then halve.
- ☐ **C** Add all four sides.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Shapes and angles, M2.5.5. Name a square, rectangle, rhombus, parallelogram and trapezium.

BY THE END YOU CAN

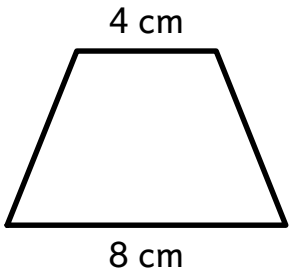
- Find the area of a trapezium
- Find the area of a shape made of rectangles and triangles

A **trapezium** is a four sided shape with one pair of **parallel sides**. Parallel sides run side by side and never meet. A flower bed with a long side and a short side can be a trapezium.

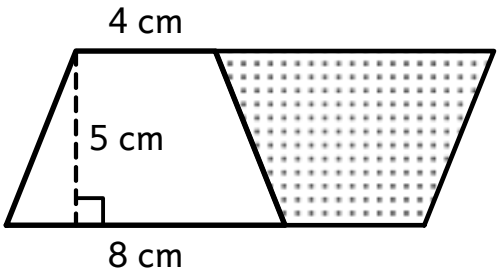
You know the area of a parallelogram. Use it. Make a second copy of the trapezium, turn it half way round, and fit it beside the first.

DRAW IT FIRST

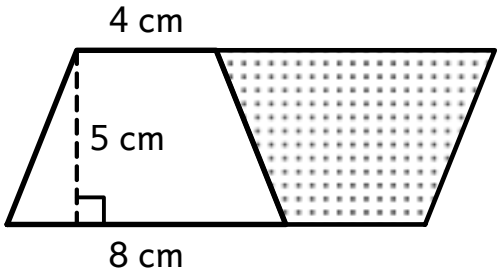
Two trapeziums make a parallelogram



1 Draw the trapezium. Its parallel sides are 8 cm and 4 cm long.



2 Add a copy, turned half way round. The height is 5 cm.



base: 8 + 4 = 12 cm

3 The two make a parallelogram. Its base is 8 + 4 = 12 cm.

The parallelogram has a base of 8 + 4 = 12 cm and a height of 5 cm. Its area is 12 × 5 = 60 cm^2. The trapezium is one of the two copies, so it is half of that.

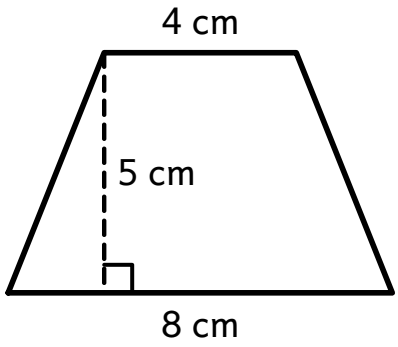
To find the area of a trapezium, add the two parallel sides. Multiply by the height. Then halve the answer.

WORKED EXAMPLE

A trapezium has parallel sides of 8 cm and 4 cm. Its height is 5 cm. What is its area?

1. Add the parallel sides: 8 + 4 = 12 cm.
2. Multiply by the height: 12 × 5 = 60 cm^2.
3. Halve it: 60 ÷ 2 = 30 cm^2.

Answer: 30 cm^2



The height is the dashed line, at a right angle to both parallel sides.

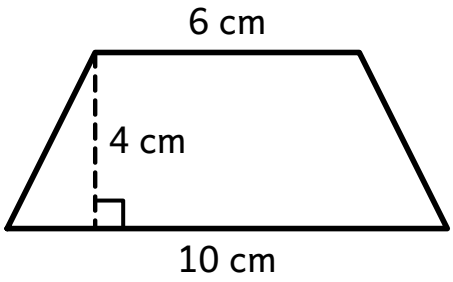
There is another way to say the same thing. Half way between the two parallel sides is (8 + 4) ÷ 2 = 6 cm. A rectangle 6 cm by 5 cm has the same area: 30 cm^2.

WORKED EXAMPLE

A trapezium has parallel sides of 10 cm and 6 cm. Its height is 4 cm. What is its area?

1. Add the parallel sides: 10 + 6 = 16 cm.
2. Multiply by the height: 16 × 4 = 64 cm^2.
3. Halve it: 64 ÷ 2 = 32 cm^2.

Answer: 32 cm^2



The parallel sides are 10 cm and 6 cm. The height is 4 cm.

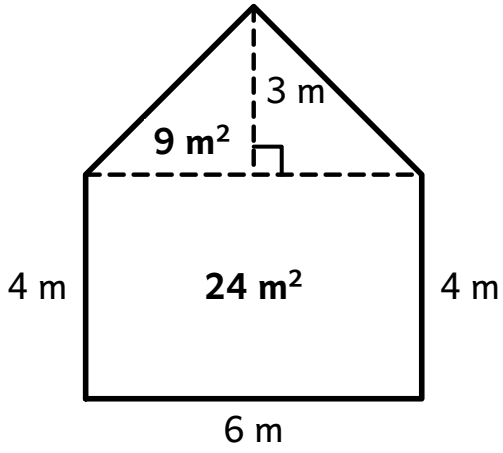
Now think of a shape made of plain parts, such as the end wall of a house. It is a rectangle with a triangle on top. A shape like this is a **joined shape**. Split it into parts you know, find each area, then add.

WORKED EXAMPLE

The end of a house is a rectangle 6 m wide and 4 m high. A triangle roof, 3 m high, sits on top. What is the area of the end?

1. Split the shape with a flat dashed line into a rectangle and a triangle.
2. The rectangle is 6 m by 4 m: $6 \times 4 = 24$ m^2.
3. The triangle has a base of 6 m and a height of 3 m. It is half of 6×3, which is 9 m^2.
4. Add the two areas: $24 + 9 = 33$ m^2.

Answer: 33 m^2



The rectangle is 24 m^2. The roof is 9 m^2. Together they make 33 m^2.

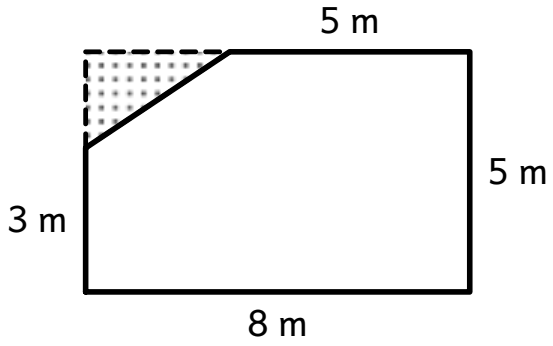
Sometimes it is quicker to take a part away. Find the whole rectangle. Then take away the corner that is missing.

WORKED EXAMPLE

A rectangle is 8 m by 5 m. A triangle is cut off one corner. The top is now 5 m long and the left side is 3 m long. What is the area left?

1. Imagine the whole rectangle. It is 8 m by 5 m, so its area is $8 \times 5 = 40$ m^2.
2. The cut corner is a triangle. Across, it is $8 - 5 = 3$ m.
3. Down, it is $5 - 3 = 2$ m.
4. The triangle is half of 3×2, which is 3 m^2.
5. Take it away: $40 - 3 = 37$ m^2.

Answer: 37 m^2



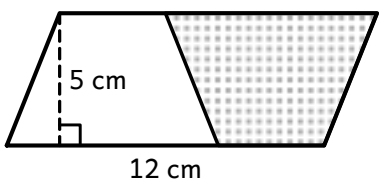
The whole rectangle is 40 m^2. The dotted corner is 3 m across and 2 m down, so it is 3 m^2. The shape is $40 - 3 = 37$ m^2.

SEE IT THREE WAYS

With things

Cut a trapezium from paper. Cut a second one the same. Turn the second half way round, flat on the table. Do not flip it over. Fit it to the first.

As a picture



In symbols

Area of the trapezium = half of (8 + 4) × 5 = half of 60 = 30 cm^2.

TRY IT

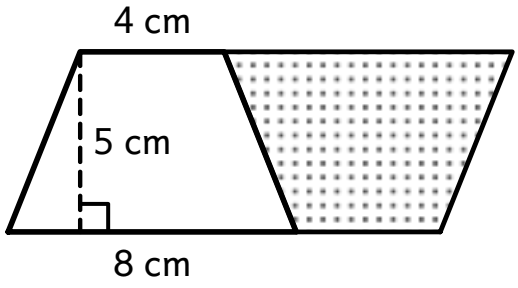
Make a parallelogram from two trapeziums

You need: two sheets of paper, scissors, a ruler, a pencil.

1. Draw a trapezium on the paper. Make the top side shorter than the bottom side. Cut it out.
2. Trace round it and cut out a copy.
3. Turn the copy half way round, flat on the table. Do not flip it over. Fit its sloping side against the same side of the first.
4. Measure the base of the new shape. Compare it with the two parallel sides added.

What you will see: The base of the new shape is the two parallel sides added together. For a trapezium with sides 8 cm and 4 cm, the base is 12 cm. Each trapezium is half of the parallelogram.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~The area of a trapezium is its longer parallel side × its height.~~

Why it feels right: The longer side looks like the base, and base × height worked for a parallelogram.

The test that breaks it: A rectangle 8 cm by 5 cm has an area of 40 cm^2. The trapezium fits inside it with room left over, because its top side is only 4 cm. So its area must be less than 40 cm^2.

What is true: Add the two parallel sides, multiply by the height, and halve. For this trapezium that is 30 cm^2.

KEY WORDS

trapezium
a four sided shape with one pair of parallel sides.

parallel sides
sides that run side by side and never meet.

joined shape
a shape made by putting rectangles and triangles together.

BACK TO YOUR GUESS

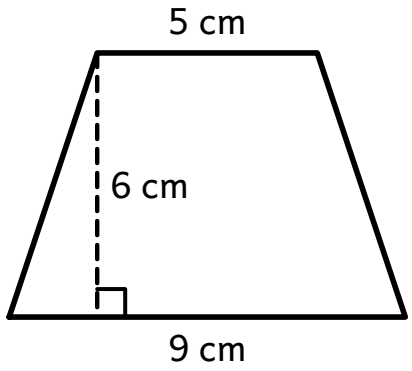
The answer is B: Add the two parallel sides, multiply by the height, then halve.

Two copies of the bed fit together to make *a* parallelogram. Its base is the two parallel sides added. Its area is that base × the height. One bed is half of it.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. What is the area of this trapezium?



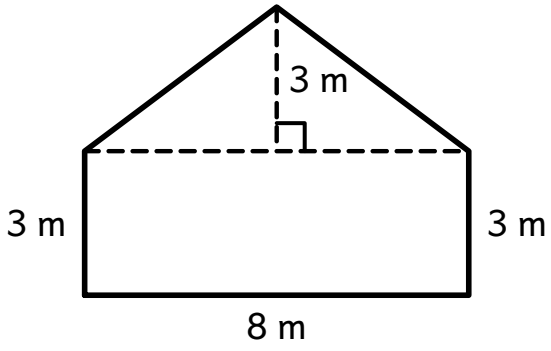
A 42 cm^2

B 54 cm^2

C 84 cm^2

Answer at the back of the book.

2. The end of a shed is a rectangle 8 m wide and 3 m high. A triangle roof, 3 m high, sits on top. What is the area of the end?



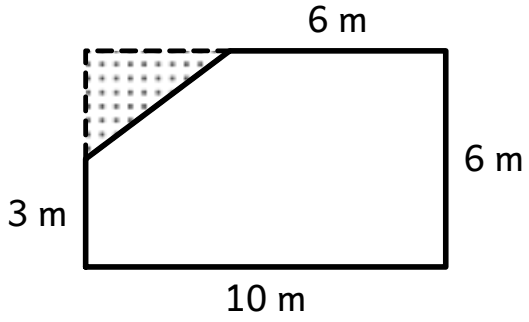
A 48 m^2

B 24 m^2

C 36 m^2

Answer at the back of the book.

3. A field is a rectangle 10 m by 6 m with a corner cut off. Find its area.



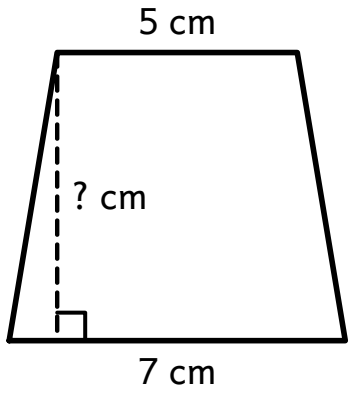
A 60 m^2

B 54 m^2

C 66 m^2

Answer at the back of the book.

4. A trapezium has parallel sides of 7 cm and 5 cm and an area of 36 cm^2. How long is its height?



A 3 cm

B 18 cm

C 6 cm

Answer at the back of the book.

5. Why do we halve at the end when we find the area of a trapezium?

A Two copies make a parallelogram, and one trapezium is half of it.

B Because a trapezium has only one pair of parallel sides.

C Because a trapezium is half of the rectangle that fits round it.

Answer at the back of the book.

6. A trapezium has parallel sides of 7 cm and 11 cm and a height of 4 cm. What is its area?

Answer at the back of the book.

7. A shape is a rectangle 6 m wide and 3 m high, with a triangle on top. The triangle has a base of 6 m and a height of 4 m. What is the area?

Answer at the back of the book.

8. **From an earlier chapter**

A parallelogram has a base of 12 cm and a height of 5 cm. What is its area?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Trapezium: add the two parallel sides, multiply by the height, then halve. Joined shape: split it into rectangles and triangles and add. Or take a missing corner away.

Draw: A trapezium with its parallel sides labelled and a dashed height. Beside it, a house end split by a flat dashed line.

M3.6.6 CHAPTER 6

Circles

How far is it all the way round a round table?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** About twice the distance across.
- ☐ **B** About four times the distance across.
- ☐ **C** A little more than three times the distance across.

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Fractions, decimals and percentages, M3.3.6. Multiply a decimal by a whole number, like 2.4 × 3.
- Level 2, Measuring, M2.6.8. Find the perimeter of a shape by adding its sides.

BY THE END YOU CAN

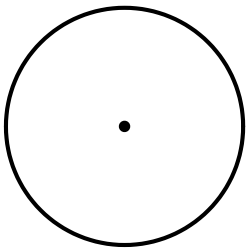
- Name the radius, diameter and circumference of a circle
- Measure round tins to see that circumference ÷ diameter is about 3.14
- Find the circumference and area of a circle

A wheel, a coin and a plate are all circles. A circle has three parts that you will use again and again.

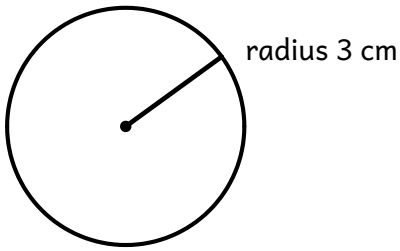
The **radius** is a straight line from the centre to the edge. The **diameter** is a straight line across, through the centre. The **circumference** is the distance all the way round the edge.

DRAW IT FIRST

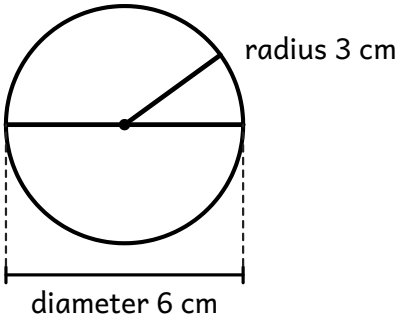
The parts of a circle



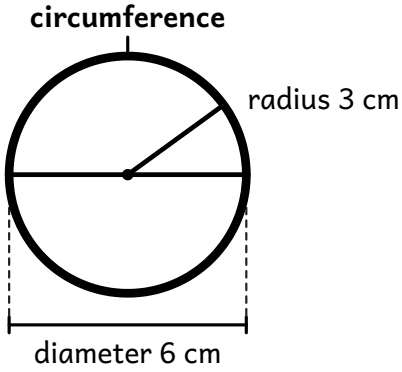
1 Draw a circle. Mark its centre with a dot.



2 The radius goes from the centre to the edge. Here it is 3 cm.



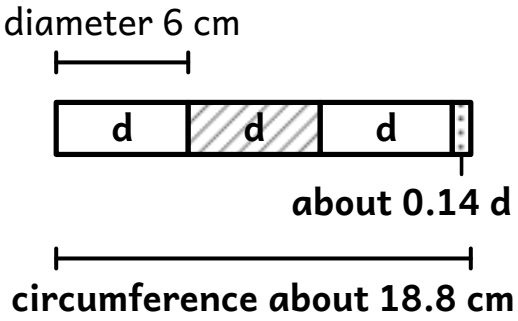
3 The diameter goes across, through the centre. It is two radii, 6 cm.



4 The circumference is the whole edge, the distance all the way round.

The diameter is always twice the radius. So a radius of 3 cm means a diameter of 6 cm. If you know the diameter, halve it to find the radius.

Now a big question. How does the circumference compare with the diameter? Imagine the edge of a circle cut open and laid out straight.



The circumference is three diameters and a little more.

This is true for every circle, big or small. The circumference divided by the diameter is always the same number. That number is called **pi**, written π.

Pi is about 3.14. We use 3.14 because it is a rounded value of π. The digits of π go on for ever, so no number written in digits is exactly π.

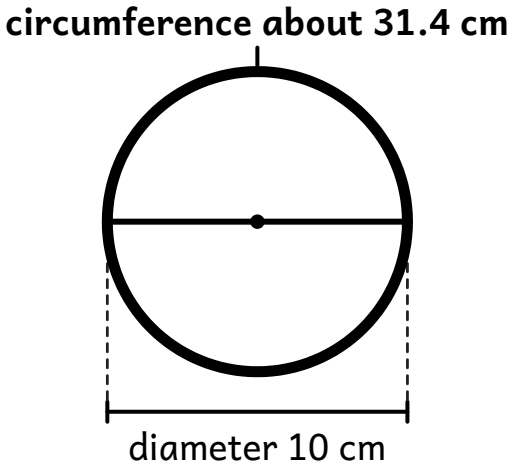
So the circumference is π × the diameter. With 3.14 for π, circumference ≈ 3.14 × diameter.

WORKED EXAMPLE

A circle has a diameter of 10 cm. What is its circumference? Use 3.14 for π.

1. The circumference is π × the diameter.
2. The diameter is 10 cm.
3. 3.14 × 10 = 31.4 cm.

Answer: 31.4 cm



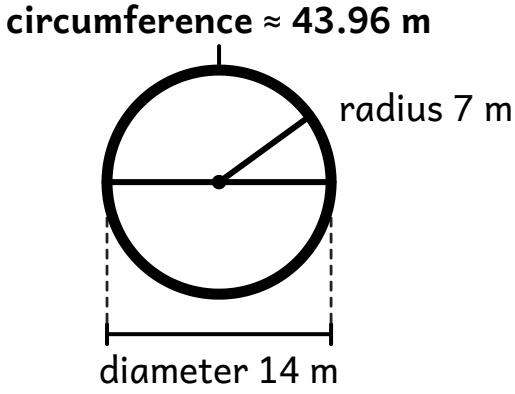
The diameter is 10 cm. Using 3.14, the circumference is about 31.4 cm.

WORKED EXAMPLE

A circle has a radius of 7 m. What is its circumference? Use 3.14 for π.

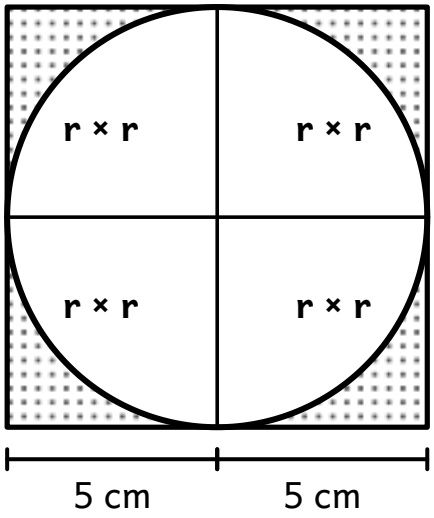
1. The radius is 7 m. The diameter is double that: 2 × 7 = 14 m.
2. The circumference is π × the diameter.
3. 3.14 × 14 = 43.96 m.

Answer: 43.96 m



The radius is 7 m. The diameter is 14 m. The circumference is about 43.96 m, from 3.14 × 14.

Now the area, the space inside a circle. Put the circle inside a square. The square has sides of two radii, so it is made of four small squares. Each small square is r by r.



The circle fills a little less than the four r × r squares.

The circle covers a little more than three of the four small squares. In fact it covers π of them, which is about 3.14. So the area of a circle is π × r × r. We write r × r as r^2.

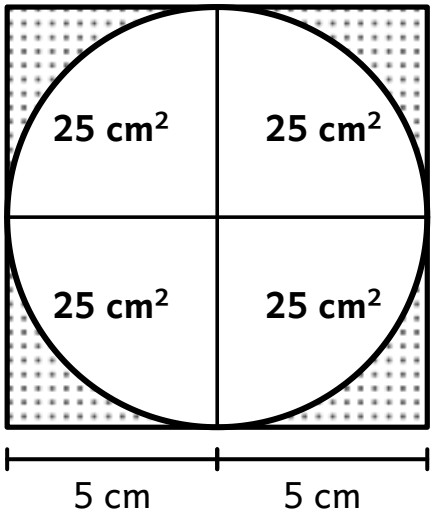
Work out r × r first. Then multiply by 3.14. Use the radius, not the diameter.

WORKED EXAMPLE

A circle has a radius of 5 cm. What is its area? Use 3.14 for π.

1. The area is π × r × r.
2. The radius is 5 cm, so r × r = 5 × 5 = 25 cm^2.
3. 3.14 × 25 = 78.5 cm^2.
4. Check: the circle is smaller than the square round it. That square is 10 cm by 10 cm, which is 100 cm^2. And 78.5 is less than 100.

Answer: 78.5 cm^2



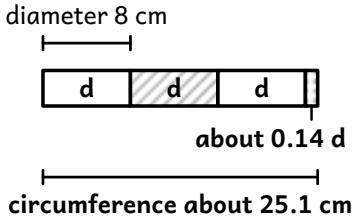
Each small square is 5 × 5 = 25 cm². The circle is 3.14 of them: 78.5 cm².

SEE IT THREE WAYS

With things

Wrap a string once round a tin. Straighten the string and measure it. Measure across the tin. Divide the first length by the second.

As a picture



In symbols

Circumference ÷ diameter ≈ 3.14. So circumference ≈ 3.14 × diameter, and area ≈ 3.14 × r × r.

TRY IT

Measure round tins

You need: three round tins or cups of different sizes, a piece of string, a ruler, a pencil.

1. Wrap the string once round the middle of a tin. Pinch the string where it meets itself.
2. Straighten the string and measure it with the ruler. This is the circumference.
3. Measure across the top of the tin, through the middle. This is the diameter.
4. Divide the circumference by the diameter. Do this for each tin and compare.

What you will see: Every answer is close to 3.1 or 3.2. It is not exactly 3.14, because string stretches and a ruler is read by eye. The answers are close to each other because the number is the same for every circle.

If you cannot try it, this is what happens.

Across the tin



7 cm

String round the tin



22 cm

22 ÷ 7 is about 3.1

A WRONG IDEA MANY PEOPLE HAVE

~~To square the radius, you double it.~~

Why it feels right: For a radius of 2, both r × r and 2 × r give 4, so they look the same.

The test that breaks it: Try a radius of 5 cm. Then r × r is 5 × 5 = 25, but 2 × r is 10. The circle holds about 78.5 cm^2, not 3.14 × 10 = 31.4 cm^2.

What is true: r^2 means r × r. Multiply the radius by itself, then multiply by π.

KEY WORDS

radius
the distance from the centre of a circle to its edge.

diameter
the distance across a circle, through its centre. It is twice the radius.

circumference
the distance all the way round a circle.

pi
the number you get from circumference ÷ diameter. It is about 3.14.

BACK TO YOUR GUESS

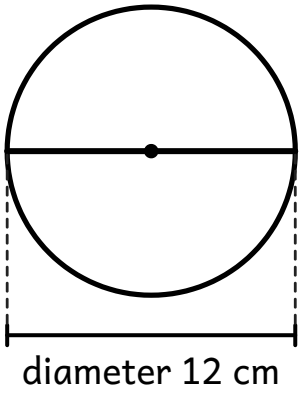
The answer is C: A little more than three times the distance across.

Go round any circle with a string, then lay the string along a line across the circle. The string is a little more than three times as long. The number is a little more than 3.14, and it is the same for every circle.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. The diameter of a circle is 12 cm. What is its radius?



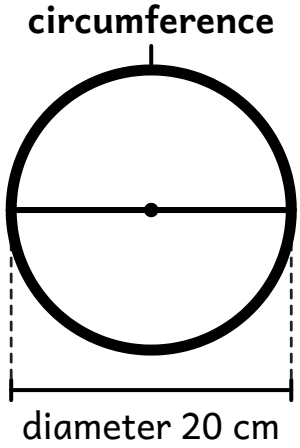
A 24 cm

B 6 cm

C 12 cm

Answer at the back of the book.

2. A circle has a diameter of 20 cm. What is its circumference? Use 3.14 for π.



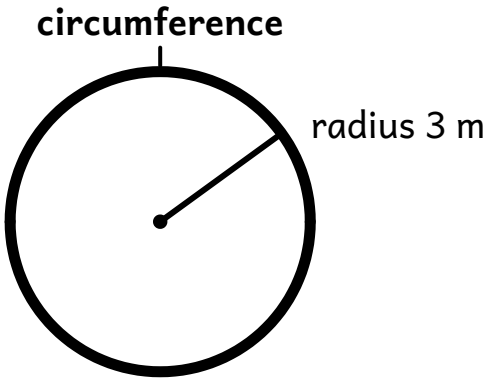
A 62.8 cm

B 31.4 cm

C 314 cm

Answer at the back of the book.

3. A circle has a radius of 3 m. What is its circumference? Use 3.14 for π.



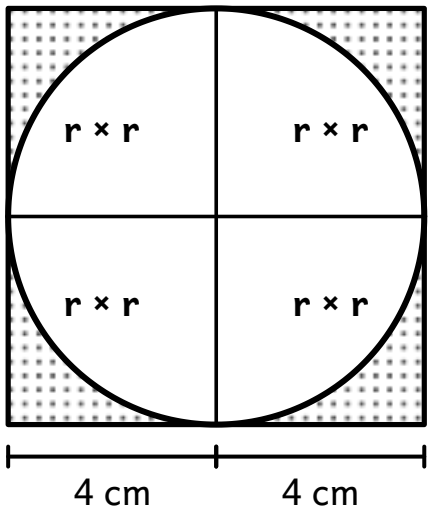
A 9.42 m

B 28.26 m

C 18.84 m

Answer at the back of the book.

4. A circle has a radius of 4 cm. What is its area? Use 3.14 for π.



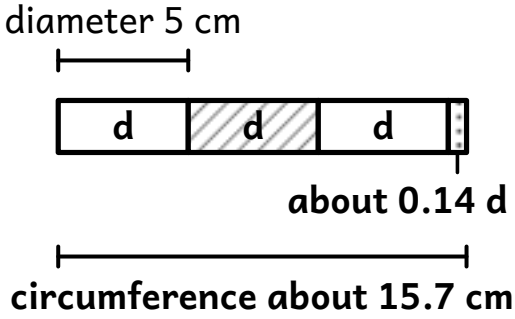
A 25.12 cm^2

B 50.24 cm^2

C 200.96 cm^2

Answer at the back of the book.

5. The edge of a circle is laid out straight and compared with the diameter. Which sentence is true?



A The circumference is a little more than three diameters.

B The circumference is exactly three diameters.

C The circumference is twice the diameter.

Answer at the back of the book.

6. A tin is 8 cm across. A string that goes once round it is 25 cm long. What is circumference ÷ diameter, to one decimal place?

A 0.3

B 3.1

C 3.14

Answer at the back of the book.

7. A circle has a diameter of 7 cm. What is its circumference? Use 3.14 for π.

Answer at the back of the book.

8. A circle has a radius of 3 cm. What is its area? Use 3.14 for π.

Answer at the back of the book.

9. **From an earlier chapter**

A square has sides of 10 cm. What is its area?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Diameter = 2 × radius. Circumference = π × diameter, about 3.14 × diameter. Area = π × r × r. π is about 3.14, a rounded value.

Draw: A circle with its centre, a radius and a diameter drawn and named. Mark the edge thick and write circumference.

M3.6.7 CHAPTER 7

Volume of cuboids

How much water fits in this tank?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Multiply the length, the width and the height.
- ☐ **B** Add the length, the width and the height.
- ☐ **C** Multiply the length by the width only.

IF THIS FEELS HARD, READ THIS FIRST

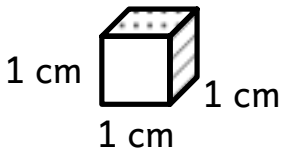
- Level 3, Angles, triangles and symmetry, M3.5.8. Draw a cube on dotted paper.
- Level 2, Measuring, M2.6.5. Say that 1 000 ml make 1 litre.

BY THE END YOU CAN

- Count cubes to find the volume of a box
- Find the volume of a cuboid as length × width × height
- Change between cm^3, ml and litres

A tank, a box and a brick are all solid shapes. A solid shape with six rectangle faces is a **cuboid**. How much space is inside one? That space is its **volume**.

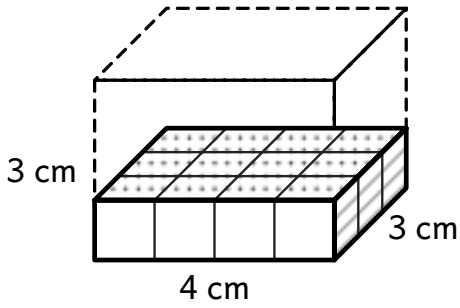
We count volume in cubes. A **cubic centimetre** is a cube with edges of 1 cm. We write it 1 cm^3. A box with a volume of 24 cm^3 holds 24 of these small cubes.



One cubic centimetre, 1 cm^3.

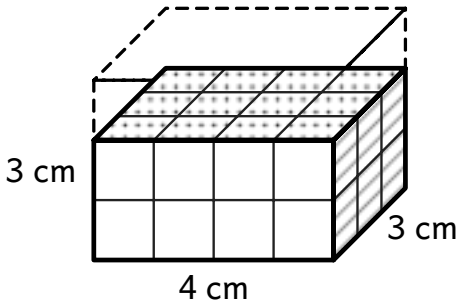
DRAW IT FIRST

Fill *a* box l*a*yer by l*a*yer



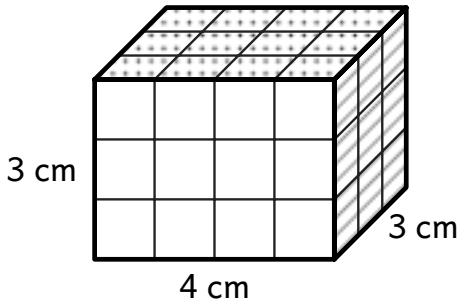
12 cm^3

1 Cover the bottom of the box with cubes. One l*a*yer holds 4 × 3 = 12 cubes.



24 cm^3

2 Add *a* second l*a*yer. Now there *a*re 2 × 12 = 24 cubes.



36 cm^3

3 Add *a* third l*a*yer. The box is full, with 3 × 12 = 36 cubes. Its volume is 36 cm^3.

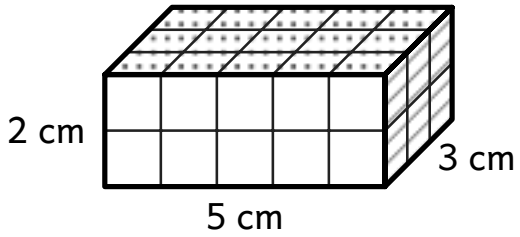
Look at the bottom layer. Its cubes are length × width. That is the same as the area of the bottom. Then the height says how many layers there are. So the volume of a cuboid is length × width × height.

WORKED EXAMPLE

A box is 5 cm long, 3 cm wide and 2 cm high. What is its volume?

1. One layer has 5 × 3 = 15 cubes.
2. The box is 2 cm high, so there are 2 layers.
3. The volume is 15 × 2 = 30 cm^3.

Answer: 30 cm^3



Two layers of 15 cubes make 30 cm^3.

The unit is cm^3 because you multiply three lengths in cm. A cube has the same length, width and height. A cube with edges of 5 cm has a volume of 5 × 5 × 5 = 125 cm^3.

Liquids are measured in ml and litres. The two measures fit together. A cube with edges of 1 cm holds 1 ml of water. So 1 cm^3 is 1 ml.

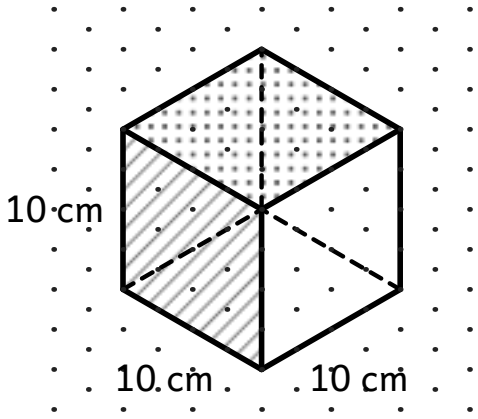
A bigger cube has edges of 10 cm. Its volume is 10 × 10 × 10 = 1 000 cm^3, which is 1 000 ml. We call 1 000 ml one **litre**.

WORKED EXAMPLE

A box is 10 cm long, 10 cm wide and 10 cm high. How much water does it hold?

1. The volume is length × width × height.
2. $10 \times 10 \times 10 = 1\,000$ cm^3.
3. 1 cm^3 holds 1 ml, so the box holds 1 000 ml.
4. And 1 000 ml is 1 litre.

Answer: 1 litre



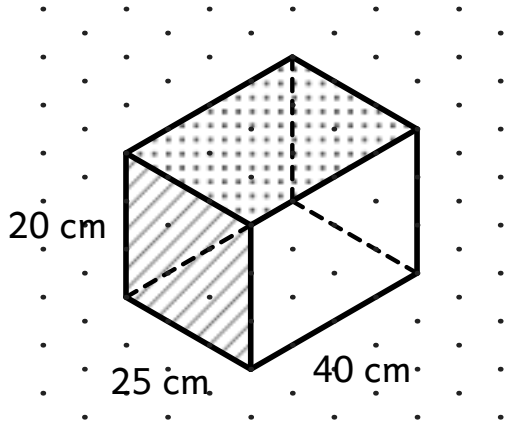
This cube is 10 cm each way. It holds 1 000 cm^3, which is 1 litre. Not to scale.

WORKED EXAMPLE

A tank is 40 cm long, 25 cm wide and 20 cm high. How many litres of water fill it?

1. The base is 40 cm by 25 cm. Its area is $40 \times 25 = 1\,000$ cm^2.
2. The tank is 20 cm high, so the volume is $1\,000 \times 20 = 20\,000$ cm^3.
3. 1 litre is 1 000 cm^3, so divide by 1 000.
4. $20\,000 \div 1\,000 = 20$ litres.

Answer: 20 litres



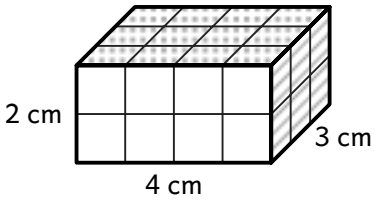
The tank holds 20 000 cm^3, which is 20 litres. Not to scale.

SEE IT THREE WAYS

With things

Stack small cubes, such as dice, into a box shape. Make it 4 across, 3 deep and 2 high. Count them all.

As a picture



In symbols

Volume = length × width × height = 4 × 3 × 2 = 24 cm^3.

TRY IT

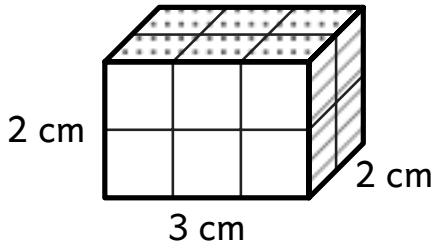
Build a box from cubes

You need: 12 small cubes of the same size, such as dice or sugar lumps, a flat table.

1. Build a box that is 3 cubes long, 2 cubes wide and 1 cube high. Count the cubes.
2. Put a second layer on top. Count again.
3. Work out 3 × 2 × 2 and compare it with your count.

What you will see: The count and the product are the same, 12. If each cube has edges of 1 cm, the volume of the box is 12 cm^3.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~**To find the volume of a box, add its length, width and height.**~~

Why it feels right: You add lengths to find a perimeter, so adding seems natural.

The test that breaks it: Take a box 4 cm by 3 cm by 2 cm. Adding gives 4 + 3 + 2 = 9. But the bottom layer alone has 4 × 3 = 12 cubes, so the box holds more than 9.

What is true: Volume counts cubes in layers. Multiply the length, the width and the height.

KEY WORDS

cuboid
a solid shape with six rectangle faces, such *as a* brick.

volume
the space inside *a* solid shape, counted in cubes.

cubic centimetre
a cube with edges of 1 cm. Written cm^3.

litre
a unit for liquids. 1 litre is 1 000 ml, which is 1 000 cm^3.

BACK TO YOUR GUESS

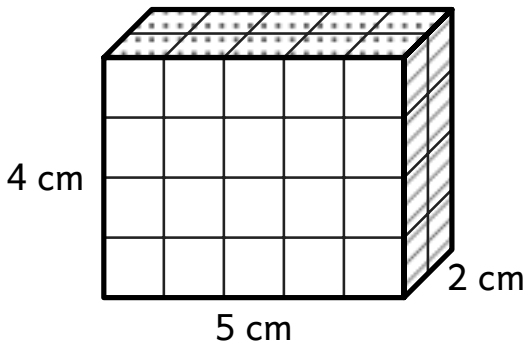
The answer is A: Multiply the length, the width and the height.

A tank fills in layers. One layer has length × width cubes of water. The height tells you how many layers there are. So you multiply all three numbers.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. What is the volume of this box?



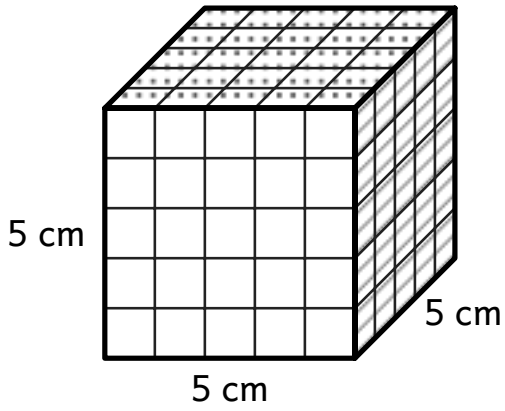
A 11 cm^3

B 40 cm^3

C 10 cm^3

Answer at the back of the book.

2. A cube has edges of 5 cm. What is its volume?



A 15 cm^3

B 25 cm^3

C 125 cm^3

Answer at the back of the book.

3. A box has a volume of 120 cm^3. Its base is 8 cm by 5 cm. How high is it?

A 3 cm

B 15 cm

C 40 cm

Answer at the back of the book.

4. A tank holds 18 000 cm^3 of water. How many litres is that?

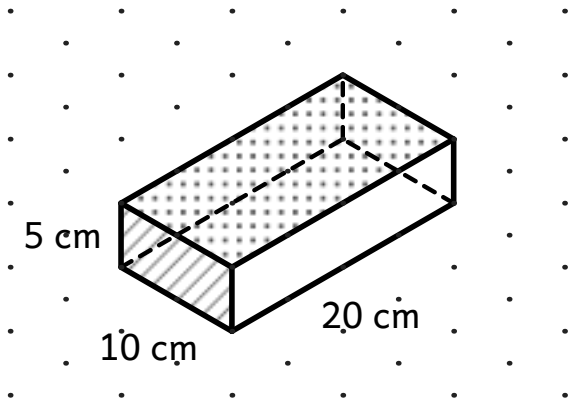
A 180 litres

B 1.8 litres

C 18 litres

Answer at the back of the book.

5. A box is 20 cm long, 10 cm wide and 5 cm high. How many millilitres of water does it hold?



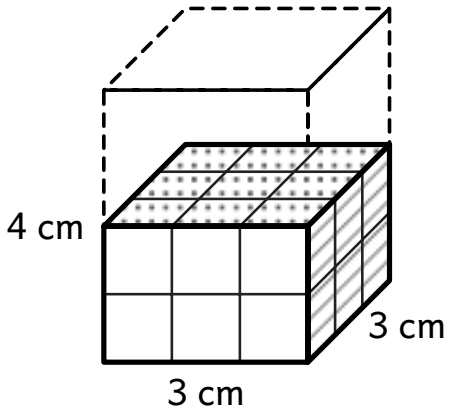
A 35 ml

B 1 000 ml

C 200 ml

Answer at the back of the book.

6. This box is 3 cm long, 3 cm wide and 4 cm high. Two layers are filled with cubes. How many cubes fill the whole box?



A 36 cubes

B 9 cubes

C 10 cubes

Answer at the back of the book.

7. A box is 6 cm long, 4 cm wide and 3 cm high. What is its volume?

Answer at the back of the book.

8. A tank holds 7 000 cm^3 of water. How many litres is that?

   Answer at the back of the book.

9. **From an earlier chapter**

   The base of a box is 6 cm by 5 cm. What is the area of the base?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Volume = length × width × height, in cm^3. 1 cm^3 = 1 ml. 1 litre = 1 000 cm^3 = 1 000 ml. A cube of 10 cm holds 1 litre.

Draw: A box of 1 cm cubes, 4 by 3 by 2, with its edges labelled. Write 4 × 3 × 2 = 24 cm^3.

M3.6.8 CHAPTER 8

Surface area

How much paper do I need to wrap this box?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** The volume of the box, length × width × height.
- ☐ **B** The area of all six faces of the box added together.
- ☐ **C** The area of one face of the box.

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Angles, triangles and symmetry, M3.5.8. Draw a solid shape and its net.
- Level 2, Shapes and angles, M2.5.7. Match a solid shape to its net.

BY THE END YOU CAN

- Find the surface area of a cuboid from its net
- Find how much paper covers a box with no gaps

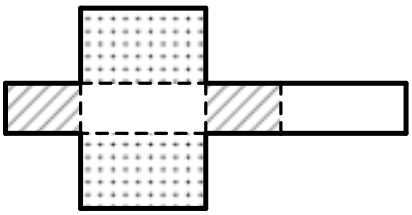
You want to wrap a box in paper with no gaps and no overlaps. The paper must cover the whole outside. The total area of all the faces of a solid is its **surface area**.

A cuboid has six **faces**, and every face is a rectangle. They come in three pairs of equal faces. The top and bottom are a pair. The front and back are a pair. The left and right are a pair.

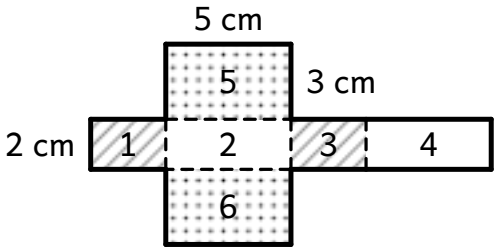
Cut a box along some edges and lay it flat. The flat shape is a **net**. Its six rectangles are the six faces. Find the area of each rectangle, then add them.

DRAW IT FIRST

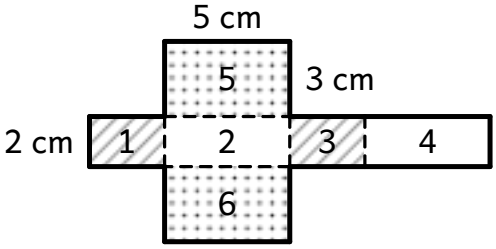
Add the six faces of a net



1 Draw the net of the box. It has six rectangles. Matching faces are shaded the same way.



2 Write the lengths on it: 5 cm, 3 cm and 2 cm.



15 + 15 + 10 + 10 + 6 + 6
= 62 cm²

3 Find the area of each face and add them. The total is 62 cm².

Faces come in pairs, so you only need three areas. Work out one face from each pair. Add the three areas. Then double the total.

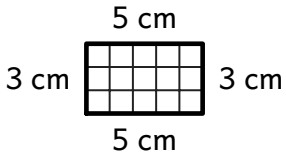
WORKED EXAMPLE

A box is 5 cm long, 3 cm wide and 2 cm high. What is its surface area?

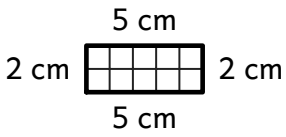
1. The top and bottom are each 5 cm by 3 cm: $5 \times 3 = 15$ cm^2.
2. The front and back are each 5 cm by 2 cm: $5 \times 2 = 10$ cm^2.
3. The left and right are each 3 cm by 2 cm: $3 \times 2 = 6$ cm^2.
4. Add one of each pair: $15 + 10 + 6 = 31$ cm^2. Then double it: $2 \times 31 = 62$ cm^2.

Answer: 62 cm^2

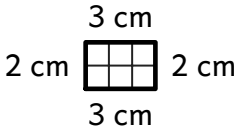
top and bottom: 15 cm^2 each



front and back: 10 cm^2 each



left and right: 6 cm^2 each



Each rectangle appears twice on the box.

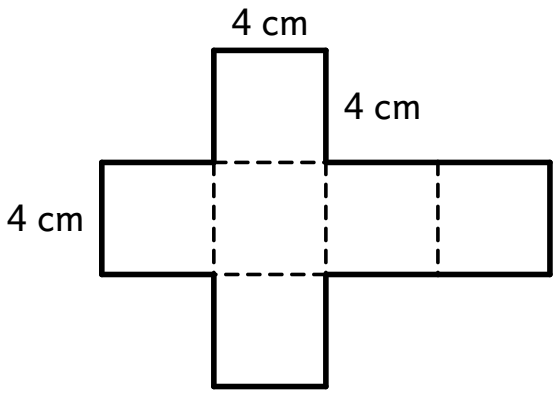
A cube is quicker, because all six faces are the same square.

WORKED EXAMPLE

A cube has edges of 4 cm. What is its surface area?

1. Every face is a square, 4 cm by 4 cm.
2. One face has an area of $4 \times 4 = 16$ cm^2.
3. A cube has 6 faces: $6 \times 16 = 96$ cm^2.

Answer: 96 cm^2



Six squares of 16 cm^2 each make 96 cm^2.

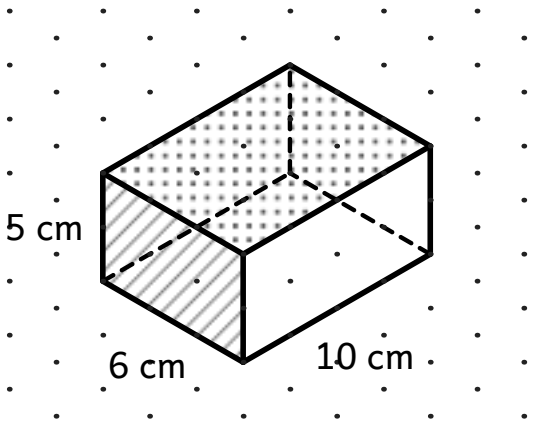
Surface area answers real questions. Paper that covers a box with no gaps is exactly its surface area. A box with no lid has one face fewer, so you take that face away.

WORKED EXAMPLE

A gift box is 10 cm long, 6 cm wide and 5 cm high. How much paper covers it with no gaps?

1. The top and bottom are 10 × 6 = 60 cm^2 each.
2. The front and back are 10 × 5 = 50 cm^2 each.
3. The left and right are 6 × 5 = 30 cm^2 each.
4. One of each pair adds to 60 + 50 + 30 = 140 cm^2. Double it: 2 × 140 = 280 cm^2.

Answer: 280 cm^2



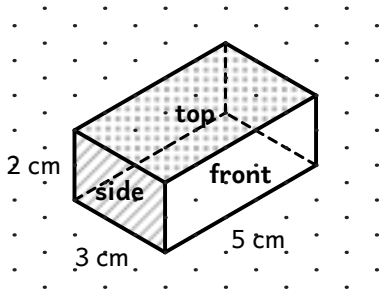
The paper for the gift box is 280 cm^2. Not to scale.

SEE IT THREE WAYS

With things

Wrap a small box in paper. Cut the paper along the edges so that it lies flat. You get six rectangles.

As a picture



In symbols

Surface area = 2 × (5 × 3) + 2 × (5 × 2) + 2 × (3 × 2) = 30 + 20 + 12 = 62 cm^2.

TRY IT

Cover *a* box with paper

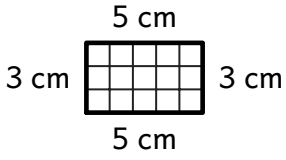
You need: *a* small box, such *as a* matchbox or *a* tea box, *a* ruler, paper, scissors, *a* pencil.

1. Measure the length, the width *and* the height of the box in cm.
2. Work out the *area* of one face from *each* pair. Add them *and* double the total.
3. Cover the box with paper, with no *gaps and* no overlaps. Cut the paper into the six faces.
4. Lay out the six pieces. Check that matching faces *are* the same size.

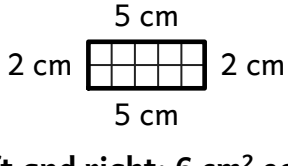
What you will see: The six pieces make three matching pairs. Their areas add up to the total you worked out. A box 5 cm by 3 cm by 2 cm needs 62 cm^2 of paper.

If you cannot try it, this is what happens.

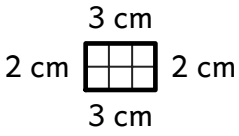
top and bottom: 15 cm^2 each



front and back: 10 cm^2 each



left and right: 6 cm^2 each



A WRONG IDEA MANY PEOPLE HAVE

~~Surface area and volume are the same thing.~~

Why it feels right: Both are about a box, and both come from its length, width and height.

The test that breaks it: A box 4 cm by 3 cm by 2 cm holds 24 cm^3. Its surface area is 52 cm^2. The numbers are different, and so are the jobs: one counts cubes inside, the other counts squares outside.

What is true: Volume is the space inside, in cm^3. Surface area is the area of the outside, in cm^2.

KEY WORDS

surface area
the total area of all the faces of a solid shape.

faces
the flat sides of a solid shape.

net
a solid shape cut along some edges and laid flat.

BACK TO YOUR GUESS

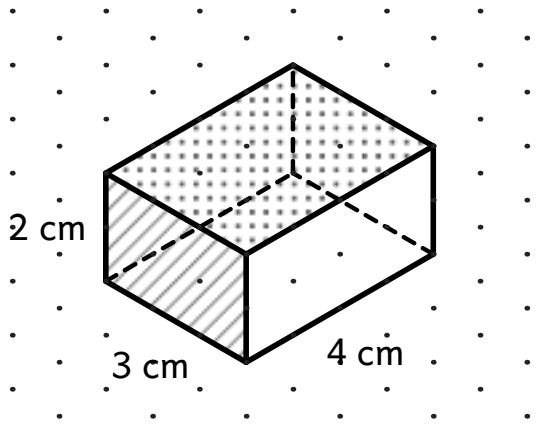
The answer is B: The area of all six faces of the box added together.

Wrapping paper must cover every face of the box. So you add the areas of all six faces. The volume counts the space inside, which is a different job.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. What is the surface area of this box?



Not to scale

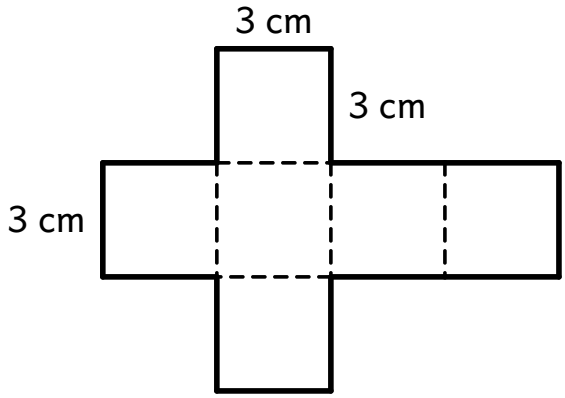
A 52 cm^2

B 24 cm^2

C 26 cm^2

Answer at the back of the book.

2. A cube has edges of 3 cm. What is its surface area?



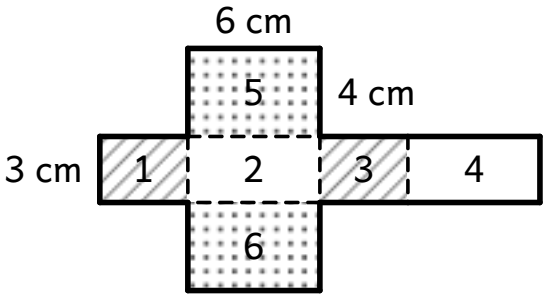
A 9 cm^2

B 27 cm^2

C 54 cm^2

Answer at the back of the book.

3. A cuboid has three different lengths. How many pairs of equal faces does it have?



A 2 pairs

B 3 pairs

C 6 pairs

Answer at the back of the book.

4. A box is 8 cm long, 5 cm wide and 3 cm high. How much paper covers it with no gaps?

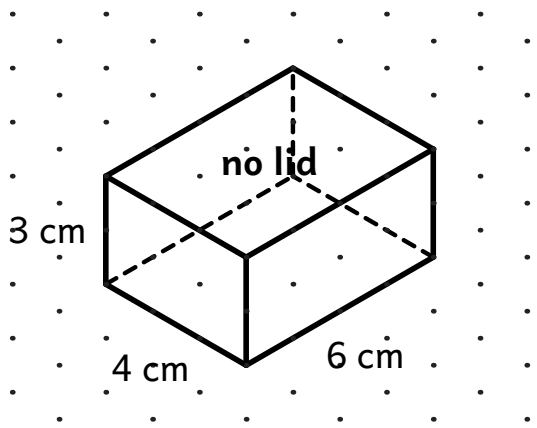
A 158 cm^2

B 79 cm^2

C 120 cm^2

Answer at the back of the book.

5. A box has no lid. It is 6 cm long, 4 cm wide and 3 cm high. How much card makes it?



Not to scale

A 108 cm^2

B 72 cm^2

C 84 cm^2

Answer at the back of the book.

6. Two boxes hold 64 cm^3 each. One is 4 by 4 by 4. The other is 8 by 4 by 2. Which needs more paper to cover it?

A The 4 by 4 by 4 box

B The 8 by 4 by 2 box

C They need the same paper.

Answer at the back of the book.

7. A box is 7 cm long, 4 cm wide and 3 cm high. What is its surface area?

Answer at the back of the book.

8. **From an earlier chapter**

A box is 5 cm long, 4 cm wide and 3 cm high. What is its volume?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A cuboid has 6 faces in 3 pairs. Surface area = 2 × (l × w + l × h + w × h). It is the paper that covers the box with no gaps, in cm^2. Volume is a different measure, in cm^3.

Draw: The net of a box with its six rectangles numbered, and the three pairs shaded alike.

Key words, A to Z

Every key word in this book, what it means, and the chapter that teaches it.

area
the space inside a shape, counted in squares. M3.6.3

base
the side of a shape that you measure the height from. M3.6.4

circumference
the distance all the way round a circle. M3.6.6

convert
to change an amount from one unit to another. M3.6.1

cubic centimetre
a cube with edges of 1 cm. Written cm^3. M3.6.7

cuboid
a solid shape with six rectangle faces, such as a brick. M3.6.7

diameter
the distance across a circle, through its centre. It is twice the radius. M3.6.6

faces
the flat sides of a solid shape. M3.6.8

height
the distance from the base straight up to the top, at a right angle to the base. M3.6.4

joined shape
a shape made by putting rectangles and triangles together. M3.6.5

litre
a unit for liquids. 1 litre is 1 000 ml, which is 1 000 cm^3. M3.6.7

metric units
units that go up and down in tens, hundreds and thousands, such as m, cm and kg. M3.6.1

missing side
a side with no length written. You work it out from the sides you know. M3.6.2

net
a solid shape cut along some edges and laid flat. M3.6.8

parallel sides
sides that run side by side and never meet. M3.6.5

parallelogram
a four sided shape with two pairs of parallel sides. M3.6.4

perimeter
the distance all the way round a shape. M3.6.2

pi
the number you get from circumference ÷ diameter. It is about 3.14. M3.6.6

radius
the distance from the centre of a circle to its edge. M3.6.6

rectangle
a four sided shape with four right angles. Its opposite sides are equal. M3.6.2

square centimetre
a square 1 cm long and 1 cm wide. Written cm^2. M3.6.3

square metre
a square 1 m long and 1 m wide. Written m^2. M3.6.3

surface area
the total area of all the faces of a solid shape. M3.6.8

tonne
a unit of mass. 1 tonne is 1 000 kg. M3.6.1

trapezium
a four sided shape with one pair of parallel sides. M3.6.5

volume
the space inside a solid shape, counted in cubes. M3.6.7

Answers

M3.6.1. Changing metric units

1. **4 200 m**

   1 km is 1 000 m. Going down the ladder you multiply: 4.2 × 1 000 = 4 200 m.

   - **If you chose A:** You multiplied by 10. Between km and m the number on the arrow is 1 000. Go to M3.6.1.
   - **If you chose B:** You multiplied by 100. That is the step from m to cm. From km to m you multiply by 1 000. Go to M3.6.1.

2. **65 cm**

   10 mm is 1 cm. Going up the ladder you divide: 650 ÷ 10 = 65 cm.

   - **If you chose B:** You divided by 100. From mm to cm the number on the arrow is 10, not 100. Go to M3.6.1.
   - **If you chose C:** You multiplied. A cm is bigger than a mm, so you need fewer cm. Go up the ladder and divide. Go to M3.6.1.

3. **2.4 kg**

   1 kg is 1 000 g. Going up the ladder you divide: 2 400 ÷ 1 000 = 2.4 kg.

   - **If you chose A:** You divided by 10. From g to kg the number on the arrow is 1 000. Go to M3.6.1.
   - **If you chose C:** You multiplied. A kg is bigger than a g, so there are fewer kg. Go up the ladder and divide. Go to M3.6.1.

4. **4.5 t**

   1 t is 1 000 kg. Going up the ladder you divide: 4 500 ÷ 1 000 = 4.5 t.

   - **If you chose A:** You divided by 100. From kg to t the number on the arrow is 1 000. Go to M3.6.1.
   - **If you chose B:** You divided by 10. From kg to t the number on the arrow is 1 000. Go to M3.6.1.

5. **1 500 m**

   Change all three to metres. 1 500 m, then 1.2 × 1 000 = 1 200 m, then 140 000 ÷ 100 = 1 400 m. The longest is 1 500 m.

   - **If you chose B:** 1.2 km is 1.2 × 1 000 = 1 200 m. That is shorter than 1 500 m. Go to M3.6.1.
   - **If you chose C:** 140 000 ÷ 100 = 1 400 m. That is shorter than 1 500 m. A bigger number does not mean a longer length when the units differ. Go to M3.6.1.

6. **It gets bigger, because a cm is smaller than a m.**

   Going down the ladder the unit gets smaller, so the number gets bigger: 5 × 100 = 500 cm.

   - **If you chose A:** A small unit fits many times into 5 m, so you count more of them. 5 m is 500 cm. Go to M3.6.1.
   - **If you chose C:** Both are lengths, but the units are different sizes. The number changes: 5 m is 500 cm. Go to M3.6.1.

7. **162 cm**

   1 m is 100 cm. So 1 m 62 cm is 100 + 62 = 162 cm.

8. **2 800 m**

   1 km is 1 000 m. So 2.8 × 1 000 = 2 800 m.

M3.6.2. Perimeter

1. **26 cm**

   Add the four sides: 9 + 4 + 9 + 4 = 26 cm. Or 2 × (9 + 4) = 26 cm.

   - **If you chose A:** You added only one length and one width. A rectangle has four sides, so add all four. Go to M3.6.2.
   - **If you chose C:** You multiplied 9 × 4. That finds the space inside, not the distance round. For the perimeter, add the sides. Go to M3.6.2.

2. **28 m**

   A square has four equal sides. 4 × 7 = 28 m.

   - **If you chose A:** You added only two sides. A square has four equal sides. Go to M3.6.2.
   - **If you chose B:** You multiplied 7 × 7. That is a different measure, the space inside. The perimeter adds the four sides. Go to M3.6.2.

3. **8 cm**

   The known sides add to 9 + 7 = 16 cm. The third side is 24 − 16 = 8 cm.

   - **If you chose B:** You added 9 + 7, the two sides you know. The third side is what is left of the perimeter after that. Go to M3.6.2.
   - **If you chose C:** You added the perimeter to the two sides. The sides are parts of the perimeter, so take them away. Go to M3.6.2.

4. **6 cm**

   The two lengths take 9 + 9 = 18 cm. That leaves 30 − 18 = 12 cm for the two widths. Each width is 12 ÷ 2 = 6 cm.

   - **If you chose A:** You took away only one length. The perimeter holds two lengths and two widths. Go to M3.6.2.
   - **If you chose C:** You took 18 from 30 and stopped. The 12 cm is for both widths together, so each width is half of it. Go to M3.6.2.

5. **40 m**

   The flat missing side is 12 − 3 = 9 m. The upright missing side is 8 − 2 = 6 m. The perimeter is 12 + 2 + 9 + 6 + 3 + 8 = 40 m.

   - **If you chose A:** You added only the four sides that have a label. The two sides marked ? are part of the perimeter too. Go to M3.6.2.
   - **If you chose B:** You multiplied 12 × 8. The perimeter is the distance round, so add the sides. The missing sides are 12 − 3 = 9 m and 8 − 2 = 6 m. Go to M3.6.2.

6. **Putting a fence round a field**

   A fence goes all the way round the edge of the field. Its length is the perimeter.

   - **If you chose B:** Tiles cover the space inside a floor. That is area, not the distance round the edge. Go to M2.6.8.
   - **If you chose C:** Sand fills the space inside a box. That is about how much a box holds, not the distance round an edge. Go to M2.6.5.

7. **38 m**

   Add the four sides: 12 + 7 + 12 + 7 = 38 m.

8. **460 cm**

   1.5 m is 1.5 × 100 = 150 cm. The perimeter is 150 + 80 + 150 + 80 = 460 cm.

M3.6.3. Area of rectangles

1. **45 cm²**

   The area is length × width: 9 × 5 = 45 cm².

   - **If you chose B:** You added 9 + 5. Area counts squares in rows, so you multiply the length by the width. Go to M3.6.3.
   - **If you chose C:** You added all four sides. That is the perimeter, the distance round. The area is length × width. Go to M3.6.2.

2. **36 m²**

   A square is a rectangle with equal sides. The area is 6 × 6 = 36 m².

   - **If you chose A:** You added 6 + 6. Multiply the two sides instead: the length and the width are both 6 m. Go to M3.6.3.
   - **If you chose C:** You added all four sides, 4 × 6. That is the perimeter. The area is 6 × 6. Go to M3.6.2.

3. **23 m²**

   Split the L with a flat line. The bottom rectangle is 7 × 2 = 14 m². The top rectangle is 3 × 3 = 9 m². Then 14 + 9 = 23 m².

   - **If you chose B:** You found the area of the whole rectangle, 7 × 5. The L has a corner missing. Take the corner away, or split the L into two rectangles. Go to M3.6.3.
   - **If you chose C:** You added all six sides. That is the perimeter. The area is the space inside. Go to M3.6.2.

4. **10 000 cm²**

   1 m is 100 cm. A square 100 cm by 100 cm has an area of 100 × 100 = 10 000 cm².

   - **If you chose A:** You used the length, 100 cm, and stopped. A square metre is 100 cm long and 100 cm wide, so multiply 100 × 100. Go to M3.6.3.
   - **If you chose B:** That is one zero too few. 100 × 100 has four zeros. Go to M3.6.3.

5. **A square with sides one centimetre long**

   Area is measured in squares. One square centimetre is the area of a square with sides one centimetre long.

   - **If you chose A:** A line one centimetre long is just a centimetre. A square centimetre is a square, so it has space inside. Go to M3.6.3.
   - **If you chose C:** A cube measures how much space a solid takes up. That is volume, which comes later. A square centimetre is flat. Go to M3.6.3.

6. **5 cm**

   Length × width = 40, so the width is 40 ÷ 8 = 5 cm. Check: 8 × 5 = 40.

   - **If you chose A:** You took 8 from 40. Area is length × width, so go back with a division. Go to M3.6.3.
   - **If you chose B:** You multiplied 40 × 8. The area is already the product, so divide it by the length instead. Go to M3.6.3.

7. **36 cm^2**

   The area is length × width: 9 × 4 = 36 cm^2.

8. **34 m**

   Add the four sides: 12 + 5 + 12 + 5 = 34 m.

M3.6.4. Area of triangles and parallelograms

1. **32 cm^2**

   The area is base × height. The base is 8 cm and the height is 4 cm. So 8 × 4 = 32 cm^2.

   - **If you chose A:** You multiplied the base by the slanted side, 8 × 5. The slanted side is not the height. Use the dashed line. Go to M3.6.4.
   - **If you chose B:** You added the four sides, 8 + 5 + 8 + 5. That is the perimeter. The area is base × height. Go to M3.6.2.

2. **30 cm^2**

   Base × height = 10 × 6 = 60 cm^2. The triangle is half of that: 60 ÷ 2 = 30 cm^2.

   - **If you chose A:** You multiplied base × height and stopped. That is the area of a parallelogram. A triangle is half of it. Go to M3.6.4.
   - **If you chose C:** You added 10 + 6. Area comes from multiplying the base and the height, then halving. Go to M3.6.4.

3. **24 cm²**

   Use the two sides at the right angle: 6 × 8 = 48 cm². The triangle is half of that: 48 ÷ 2 = 24 cm².

   - **If you chose A:** You used the slanted side, 10, with 8 and halved. The base and height are the two sides at the right angle, 6 and 8. Go to M3.6.4.
   - **If you chose B:** You multiplied 6 × 8 and stopped. The triangle is half of that rectangle. Go to M3.6.4.

4. **4 cm**

   Base × height = 36 cm², so the height is 36 ÷ 9 = 4 cm. Check: 9 × 4 = 36 cm².

   - **If you chose B:** The 5 cm side is slanted. The height is the dashed line at a right angle to the base. Go to M3.6.4.
   - **If you chose C:** You took 9 from 36. Area is base × height, so go back with a division. Go to M3.6.4.

5. **They have the same area.**

   Both have a base of 6 cm and a height of 4 cm. Each has an area of base × height, so they are equal.

   - **If you chose A:** Cut a triangle off the parallelogram and slide it to the other end. You make a rectangle that is 6 cm by 4 cm. So the two are equal. Go to M3.6.4.
   - **If you chose C:** It looks longer because it leans. But its height is still 4 cm. The slanted sides do not add area. Go to M3.6.4.

6. **6 cm**

   Two copies make a parallelogram of 2 × 24 = 48 cm². Its base is 8 cm, so the height is 48 ÷ 8 = 6 cm.

   - **If you chose B:** You divided 24 by 8. But 24 is only half of base × height, so 8 × height = 48 cm². Go to M3.6.4.
   - **If you chose C:** You halved 24. A triangle is half of base × height. So double the area first, to 48. Then divide by 8. Go to M3.6.4.

7. **20 cm²**

   Base × height = 10 × 4 = 40 cm². The triangle is half of that: 40 ÷ 2 = 20 cm².

8. **66 cm²**

   The area of a rectangle is length × width: 11 × 6 = 66 cm².

M3.6.5. Trapeziums and joined shapes

1. **42 cm²**

   Add the parallel sides: 9 + 5 = 14 cm. Multiply by the height: 14 × 6 = 84 cm². Halve it: 84 ÷ 2 = 42 cm².

   - **If you chose B:** You multiplied the longer parallel side by the height, 9 × 6. The top side is shorter, so that gives too much. Add both parallel sides and halve. Go to M3.6.5.
   - **If you chose C:** You added the parallel sides and multiplied by the height, 14 × 6. That is the area of the parallelogram made of two trapeziums. Halve it. Go to M3.6.5.

2. **36 m²**

   The rectangle is 8 × 3 = 24 m². The roof is half of 8 × 3, which is 12 m². Then 24 + 12 = 36 m².

   - **If you chose A:** You counted the roof as 8 × 3 = 24 m². A triangle is only half of base × height, so the roof is 12 m². Go to M3.6.4.
   - **If you chose B:** That is only the rectangle, 8 × 3. The roof triangle has area too. Add it. Go to M3.6.5.

3. **54 m²**

   The whole rectangle is 10 × 6 = 60 m². The cut corner is 10 − 6 = 4 m across and 6 − 3 = 3 m down. Its area is half of 4 × 3, which is 6 m². Then 60 − 6 = 54 m².

   - **If you chose A:** That is the whole rectangle, 10 × 6. The corner has been cut off, so take its area away. Go to M3.6.5.
   - **If you chose C:** You added the corner. The corner is cut off, so it is missing from the shape. Take it away. Go to M3.6.5.

4. **6 cm**

   Two copies make a parallelogram with an area of 2 × 36 = 72 cm². Its base is 7 + 5 = 12 cm. So the height is 72 ÷ 12 = 6 cm.

   - **If you chose A:** You divided 36 by 12. But 36 is only half of (7 + 5) × height. Double it first: 72, then divide by 12. Go to M3.6.5.
   - **If you chose B:** You halved 36. The trapezium is half of the parallelogram, so double the area first, then divide by the base, 12. Go to M3.6.5.

5. **Two copies make a parallelogram, and one trapezium is half of it.**

   The parallelogram has an area of (the two parallel sides added) × height. It holds two equal trapeziums. So one trapezium is half.

   - **If you chose B:** The number of parallel sides tells you the name of the shape. It is not the reason for halving. Go to M3.6.5.
   - **If you chose C:** Take the trapezium with sides 8 cm and 4 cm and height 5 cm. The rectangle round it is 8 × 5 = 40 cm^2. Half of that is 20 cm^2, but the trapezium is 30 cm^2. We halve because two copies make a parallelogram. Go to M3.6.5.

6. **36 cm^2**

   Add the parallel sides: 7 + 11 = 18 cm. Multiply by the height: 18 × 4 = 72 cm^2. Halve it: 72 ÷ 2 = 36 cm^2.

7. **30 m^2**

   The rectangle is 6 × 3 = 18 m^2. The triangle is half of 6 × 4, which is 12 m^2. Then 18 + 12 = 30 m^2.

8. **60 cm^2**

   The area of a parallelogram is base × height: 12 × 5 = 60 cm^2.

M3.6.6. Circles

1. **6 cm**

   The diameter is twice the radius, so the radius is half the diameter: 12 ÷ 2 = 6 cm.

   - **If you chose A:** You doubled the diameter. The radius is only half way across, so halve the diameter. Go to M3.6.6.
   - **If you chose C:** That is the diameter. The radius goes from the centre to the edge, which is half of the way across. Go to M3.6.6.

2. **62.8 cm**

   The circumference is 3.14 × the diameter: 3.14 × 20 = 62.8 cm.

   - **If you chose B:** You used 3.14 × 10, the radius. The circumference uses the whole diameter, 20 cm. Go to M3.6.6.
   - **If you chose C:** You worked out 3.14 × 10 × 10. That is the area rule, and it uses the radius twice. The circumference is 3.14 × the diameter. Go to M3.6.6.

3. **18.84 m**

   The diameter is 2 × 3 = 6 m. The circumference is 3.14 × 6 = 18.84 m.

   - **If you chose A:** You used 3.14 × 3, the radius. The rule needs the diameter, which is twice the radius. Go to M3.6.6.
   - **If you chose B:** You worked out 3.14 × 3 × 3. That is the area rule. The circumference is 3.14 × the diameter. Go to M3.6.6.

4. **50.24 cm^2**

   The area is 3.14 × r × r. Here r × r = 4 × 4 = 16 cm^2, and 3.14 × 16 = 50.24 cm^2.

   - **If you chose A:** You worked out 3.14 × 4 × 2. The rule needs r × r, which is 4 × 4, not 4 × 2. Go to M3.6.6.
   - **If you chose C:** You used the diameter, 8 cm, twice. The area rule uses the radius, 4 cm. Go to M3.6.6.

5. **The circumference is a little more than three diameters.**

   Three whole diameters and a short bit of about 0.14 of a diameter make the edge. That is about 3.14 diameters.

   - **If you chose B:** Look at the short bar at the end. It is not zero. The edge is three diameters and a bit more. Go to M3.6.6.
   - **If you chose C:** Twice the diameter is only two bars. The picture needs more than three bars to cover the edge. Go to M3.6.6.

6. **3.1**

   25 ÷ 8 = 3.125, which is 3.1 to one decimal place. It is close to 3.14, but not exactly, because measuring is never exact.

   - **If you chose A:** You divided 8 by 25, the width by the string. Divide the string by the width. Go to M3.6.6.
   - **If you chose C:** That is the rounded value of π. The question asks what this tin gives: 25 ÷ 8 = 3.125, about 3.1. Go to M3.6.6.

7. **21.98 cm**

   The circumference is 3.14 × the diameter: 3.14 × 7 = 21.98 cm.

8. **28.26 cm^2**

   The area is 3.14 × r × r. Here r × r = 3 × 3 = 9 cm^2, and 3.14 × 9 = 28.26 cm^2.

9. **100 cm^2**

   The area of a square is side × side: 10 × 10 = 100 cm^2.

M3.6.7. Volume of cuboids

1. **40 cm^3**

   One layer has 5 × 2 = 10 cubes. There are 4 layers. So the volume is 10 × 4 = 40 cm^3.

   - **If you chose A:** You added 5 + 2 + 4. Volume counts cubes in layers, so you multiply. Go to M3.6.7.
   - **If you chose C:** That is 5 × 2, only the bottom layer. The box is 4 layers high, so multiply by 4 as well. Go to M3.6.7.

2. **125 cm^3**

   A cube has the same length, width and height. 5 × 5 × 5 = 125 cm^3.

   - **If you chose A:** You added 5 + 5 + 5. Multiply the three edges instead. Go to M3.6.7.
   - **If you chose B:** That is 5 × 5, only one layer. The cube is 5 layers high. Go to M3.6.7.

3. **3 cm**

   One layer is 8 × 5 = 40 cubes. The volume is 120 cubes, so the height is 120 ÷ 40 = 3 cm.

   - **If you chose B:** You divided 120 by 8 and stopped. The base is 8 × 5 = 40 cm^2, so divide 120 by 40. Go to M3.6.7.
   - **If you chose C:** That is the area of the base, 8 × 5. The height is how many layers of 40 cubes make 120. Go to M3.6.7.

4. **18 litres**

   1 litre is 1 000 cm^3. So 18 000 ÷ 1 000 = 18 litres.

   - **If you chose A:** You divided by 100. A litre is 1 000 cm^3, so divide by 1 000. Go to M3.6.7.
   - **If you chose B:** You divided by 10 000. A litre is 1 000 cm^3, which has three zeros. Go to M3.6.7.

5. **1 000 ml**

   The volume is 20 × 10 × 5 = 1 000 cm^3. Each cm^3 holds 1 ml, so the box holds 1 000 ml.

   - **If you chose A:** You added 20 + 10 + 5. Volume counts cubes in layers, so you multiply. Go to M3.6.7.
   - **If you chose C:** That is 20 × 10, one layer of cubes. The box is 5 cm high, so there are 5 layers. Go to M3.6.7.

6. **36 cubes**

   One layer has 3 × 3 = 9 cubes. The full box has 4 layers. So 9 × 4 = 36 cubes.

   - **If you chose B:** That is one layer, 3 × 3. The box has 4 layers when it is full. Go to M3.6.7.
   - **If you chose C:** You added 3 + 3 + 4. Multiply the layer, 9 cubes, by the 4 layers. Go to M3.6.7.

7. **72 cm^3**

   The volume is length × width × height: 6 × 4 × 3 = 72 cm^3.

8. **7 litres**

   1 litre is 1 000 cm^3. So 7 000 ÷ 1 000 = 7 litres.

9. **30 cm^2**

   The area of a rectangle is length × width: 6 × 5 = 30 cm^2.

M3.6.8. Surface area

1. **52 cm^2**

   One of each pair: 4 × 3 = 12 cm^2. 4 × 2 = 8 cm^2. 3 × 2 = 6 cm^2. They add to 26 cm^2. Double it: 2 × 26 = 52 cm^2.

   - **If you chose B:** You multiplied 4 × 3 × 2. That is the volume, the space inside, and it is in cm^3. Surface area adds the areas of the faces. Go to M3.6.7.
   - **If you chose C:** You added one face from each pair: 12 + 8 + 6. Every face appears twice on the box, so double it. Go to M3.6.8.

2. **54 cm²**

   One face is 3 × 3 = 9 cm². A cube has six equal faces: 6 × 9 = 54 cm².

   - **If you chose A:** That is one face. A cube has six faces. Go to M3.6.8.
   - **If you chose B:** You multiplied 3 × 3 × 3. That is the volume, in cm³. Surface area adds the areas of the six faces. Go to M3.6.7.

3. **3 pairs**

   A cuboid has six faces. Opposite faces match, so they make 6 ÷ 2 = 3 pairs.

   - **If you chose A:** Count again. The top and bottom match, the front and back match, and the left and right match. That is more than two. Go to M3.6.8.
   - **If you chose C:** There are six faces in all. Two faces make a pair, so there are fewer pairs than faces. Go to M3.6.8.

4. **158 cm²**

   One of each pair: 8 × 5 = 40 cm². 8 × 3 = 24 cm². 5 × 3 = 15 cm². They add to 79 cm². Double it: 2 × 79 = 158 cm².

   - **If you chose B:** You added one face from each pair: 40 + 24 + 15. Every face appears twice, so double it. Go to M3.6.8.
   - **If you chose C:** You multiplied 8 × 5 × 3. That is the volume of the box, not the paper. Go to M3.6.7.

5. **84 cm²**

   With a lid the box needs 2 × (24 + 18 + 12) = 108 cm². The lid is 6 × 4 = 24 cm². Without it: 108 − 24 = 84 cm².

   - **If you chose A:** That covers a box with a lid. This box has no lid, so take the top face away. Go to M3.6.8.
   - **If you chose B:** You multiplied 6 × 4 × 3. That is the volume of the box. Card for the faces is an area. Go to M3.6.7.

6. **The 8 by 4 by 2 box**

   The 4 by 4 by 4 box has 2 × (16 + 16 + 16) = 96 cm². The 8 by 4 by 2 box has 2 × (32 + 16 + 8) = 112 cm². So the flatter box needs more paper.

   - **If you chose A:** The cube has 96 cm² of faces. The long flat box has 112 cm². The same volume can have a different surface area. Go to M3.6.8.
   - **If you chose C:** The same volume does not mean the same surface area. Work out the six faces of each box and compare. Go to M3.6.8.

7. **122 cm²**

   One of each pair: 7 × 4 = 28 cm². 7 × 3 = 21 cm². 4 × 3 = 12 cm². They add to 61 cm². Double it: 2 × 61 = 122 cm².

8. **60 cm³**

   The volume is length × width × height: 5 × 4 × 3 = 60 cm³.

About this book

For learners who can find the area of a shape on squared paper by counting squares, and who multiply and divide whole numbers on paper. In many schools this fits children of about 11 to 13, and older learners who never met area and volume can start here too. Each chapter takes one or two short sessions. A learner needs a ruler, squared paper, a pencil, and for some activities string, a tin and a pair of scissors. Every exercise has its answer and the reason for it. Each chapter also has a short page for the adult, with three questions to ask.

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