●●●○○ Level 3 · Mathematics M3.1

Negative numbers, factors and powers

Go below zero, break numbers into factors and primes, and write repeated multiplying as powers.

Read temperatures below zero and calculate with negative numbers. Then find factors, multiples and primes, and write repeated multiplying as squares, cubes and powers of ten.

YOU ARE READY FOR THIS BOOK IF

You can write out the 7 and 8 times tables without help.

One step up	M4.1 Powers, roots, surds and standard form
This book	**M3.1 Negative numbers, factors and powers**
One step down	M2.1 Big numbers and place value

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M3.1.5 Prime numbers

M3.1.6 Prime factors, common factors and multiples

M3.1.7 Squares, cubes and roots

M3.1.8 Powers of ten

Key words, A to Z

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M3.1.1 CHAPTER 1

Below zero

The weather report says minus 4. How can a temperature be below zero?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** A temperature of −4 °C is warmer than 0 °C.
- ☐ **B** A temperature of −4 °C is colder than 0 °C.
- ☐ **C** A temperature of −4 °C means there is no heat at all.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Big numbers and place value, M2.1.4. If the signs < and > are new to you, read this first.

BY THE END YOU CAN

- Read temperatures in °C above and below zero on a thermometer
- Order numbers like −5, 2, −1 and 0 on a number line

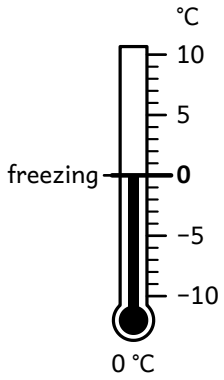
Amina looks at the thermometer outside her door on a cold morning. The liquid is below the 0 mark. The reading is minus 4.

A **thermometer** measures how hot or cold something is. In a tube thermometer the liquid rises when it is warm and falls when it is cold.

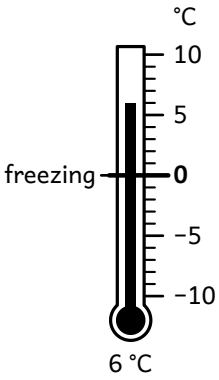
We count temperature in **degrees Celsius**, written °C. Pure water freezes at 0 °C. Marks above 0 are warmer. Marks below 0 are colder.

DRAW IT FIRST

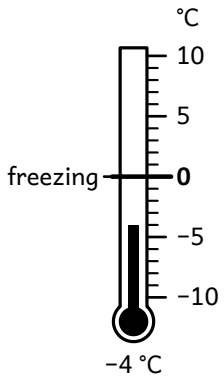
Read a thermometer from the freezing mark



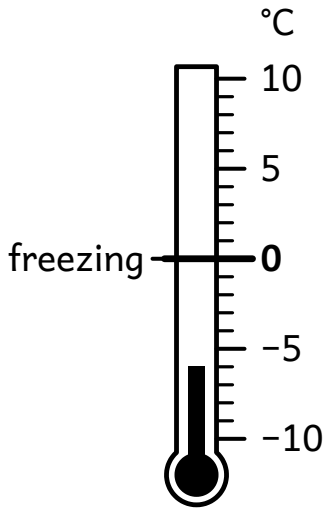
1 Find the mark 0. Pure water freezes at this temperature.



2 Liquid above 0 shows a day warmer than freezing. This liquid stops at 6 °C.



3 Liquid below 0 shows a day colder than freezing. This liquid stops 4 marks below 0, at −4 °C.



The liquid stands 6 marks below the 0 mark. Each mark is 1 degree.

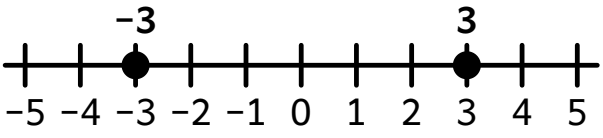
WORKED EXAMPLE

The liquid stands 6 marks below the 0 mark. Each mark is 1 degree. What does the thermometer read?

1. Each mark is 1 degree, so 6 marks is 6 degrees.
2. The liquid is below 0, so the number has a minus sign.
3. 6 marks below 0 is −6 °C.

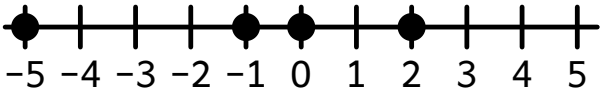
Answer: −6 °C

A number above zero is a **positive number**. A number below zero is a **negative number**. We write a minus sign in front, as in −4. Say "minus four" or "negative four". Zero is neither positive nor negative.



Negative numbers are left of 0. Positive numbers are right of 0.

A thermometer is a number line stood on its end. Up is the same as right, and down is the same as left. The further left a number is, the smaller it is. So −5 < −1 < 0 < 2.



The four numbers from the worked example, marked on a number line.

WORKED EXAMPLE

Put −5, 2, −1, 0 in order, smallest first.

1. Mark the four numbers on a number line. The one furthest left is −5.
2. The next one to the right is −1. Then comes 0, and last, on the right, is 2.
3. Read them from left to right: −5, −1, 0, 2.

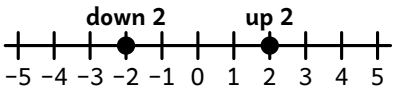
Answer: −5, −1, 0, 2

SEE IT THREE WAYS

With things

A lift in a tall building has buttons for floors above and below the ground floor. The ground floor is 0. Press 2 to go up two floors. Press −2 to go down two floors.

As a picture



In symbols

2 is above 0. The number −2 is below 0. So −2 < 0 < 2.

TRY IT

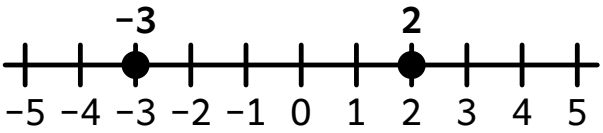
Make a number line

You need: a sheet of paper, a pencil, two small stones or buttons.

1. Draw a straight line across the paper.
2. Mark 0 near the middle.
3. Mark 1, 2, 3, 4 and 5 to the right of 0. Leave the same gap each time.
4. Mark −1, −2, −3, −4 and −5 to the left of 0, with the same gap.
5. Put one stone on 2 and one stone on −3. Which stone is on the smaller number?

What you will see: The stone on −3 is to the left of the stone on 2. The number on the left is always the smaller one.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~Minus 7 is bigger than minus 2, because 7 is bigger than 2.~~

Why it feels right: When you count 1, 2, 3, a bigger digit means a bigger number. The minus sign looks like a small mark in front.

The test that breaks it: Put both on a thermometer. Find −7 °C and −2 °C. Which one is lower down? Which one is colder?

What is true: Below zero, a bigger digit means further below zero. So −7 is smaller than −2.

KEY WORDS

thermometer
a tool that measures temperature.

degrees Celsius
the units of temperature, written °C. Water freezes at 0 °C.

positive number
a number above zero.

negative number
a number below zero, written with a minus sign.

BACK TO YOUR GUESS

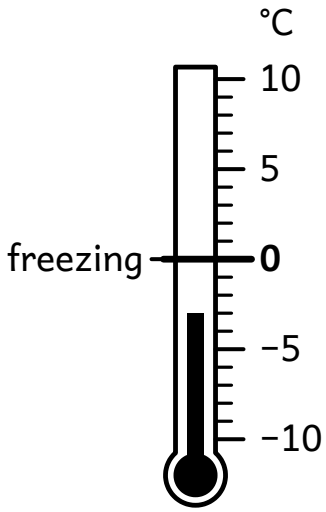
The answer is B: A temperature of −4 °C is colder than 0 °C.

Below 0 °C means colder than 0 °C. There is still heat. Zero is only the mark where we start counting, and the numbers go on below it.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Kofi reads this thermometer. What temperature does it show?



A 3 °C

B −4 °C

C −3 °C

Answer at the back of the book.

2. Which list has the numbers 3, −2, −7, 0, −1 in order, smallest first?

   A −7, −1, −2, 0, 3

   B −7, −2, −1, 0, 3

   C 3, 0, −1, −2, −7

   Answer at the back of the book.

3. Which temperature is colder, −8 °C or −3 °C?

   A −8 °C

   B −3 °C

   C 8 °C

   Answer at the back of the book.

4. Which of these numbers is smaller than −2?

   A −1

   B 0

   C −5

   Answer at the back of the book.

5. Which of these temperatures is below the freezing point of water?

   A 0 °C

   B −1 °C

   C 5 °C

   Answer at the back of the book.

6. Mei stands on the number 1. She walks 3 steps to the left. Which number is she on?

   Answer at the back of the book.

7. Which is colder, −3 °C or −6 °C? Type the colder temperature.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A number below zero is negative. Negative numbers sit to the left of 0 on a number line. The further left a number is, the smaller it is. So −5 < −1 < 0 < 2.

Draw: A number line from −5 to 5 with 0 in the middle. Mark −5, −1, 0 and 2, and write −5 < −1 < 0 < 2 under it.

M3.1.2 CHAPTER 2

Calculating with negative numbers

It is −3 °C at night and 5 °C at noon. How many degrees did it rise?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** The rise is 2 degrees, because 5 − 3 = 2.
- ☐ **B** The rise is 8 degrees. It goes from −3 up to 0, and then on up to 5.
- ☐ **C** The rise is 5 degrees, because the noon temperature is 5.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.6. If divide facts such as 12 ÷ 3 = 4 are not clear, read this first.

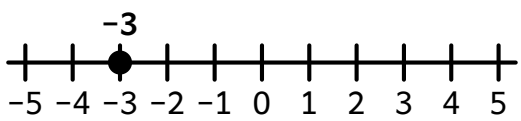
BY THE END YOU CAN

- Find the rise in temperature from −3 °C to 5 °C
- Work out −4 + 7 and 2 − 6 on a number line
- Multiply and divide with negatives, like −3 × −4

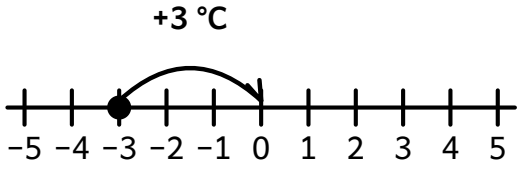
At night the temperature is −3 °C. At noon it is 5 °C. The **rise** is how many degrees it went up. How big is the rise?

DRAW IT FIRST

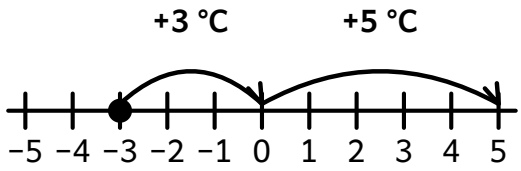
Count the rise from −3 to 5



1 Put a dot on −3. This is the night temperature.



2 Hop up to 0. That is a hop of 3.



3 Hop on from 0 up to 5. That is a hop of 5. The whole rise is 3 + 5 = 8 degrees.

WORKED EXAMPLE

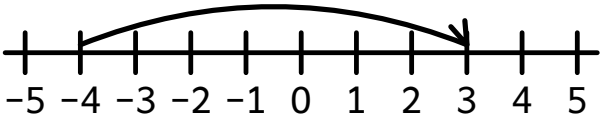
The temperature rises from −3 °C to 5 °C. How many degrees is the rise?

1. From −3 up to 0 is 3 degrees.
2. From 0 up to 5 is 5 degrees.
3. Add the two hops: 3 + 5 = 8.

Answer: 8 degrees

You can add and take away on a number line. To add a positive number, hop to the right. To take away a positive number, hop to the left. Adding a negative number moves you left, so 3 + −6 is the same as 3 − 6.

+7



Start at −4 and hop 7 to the right. You land on 3.

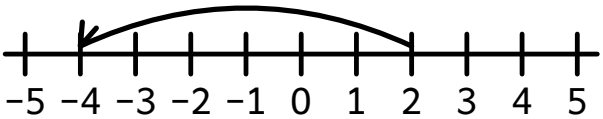
WORKED EXAMPLE

Work out −4 + 7.

1. Start at −4. Adding 7 means 7 hops to the right.
2. The first 4 hops reach 0. Then 3 more hops reach 3.
3. So −4 + 7 = 3.

Answer: −4 + 7 = 3

−6



Start at 2 and hop 6 to the left. You land on −4.

WORKED EXAMPLE

Work out 2 − 6.

1. Start at 2. Taking away 6 means 6 hops to the left.
2. Two of the hops take you from 2 to 0.
3. The other four hops go on from 0 to −4.
4. So 2 − 6 = −4.

Answer: 2 − 6 = −4

Now multiplying. 3 × −4 means 3 lots of −4. That is −4 + −4 + −4, which is −12. A positive number times a negative number gives a negative answer.

What about −3 × −4? You cannot lay out "minus 3 lots" with beans. So look for a pattern, and let it carry on.

sum	answer
3 × −4	−12
2 × −4	−8
1 × −4	−4
0 × −4	0
−1 × −4	4
−2 × −4	8
−3 × −4	12

Each row has one less lot of −4. Each answer goes up by 4.

Look down the answers: −12, −8, −4, 0. They go up by 4 each time. The pattern cannot stop at 0. The next answers are 4, 8 and 12. So −3 × −4 = 12.

The rule: two numbers with the same sign give a positive answer. Two numbers with different signs give a negative answer. This holds for the **product** of any two numbers, and for dividing too.

Dividing undoes multiplying. We know 3 × −4 = −12. So −12 ÷ 3 = −4, and −12 ÷ −4 = 3. The same sign rule works: same signs give a positive answer, and different signs give a negative answer.

times fact	divide fact
3 × −4 = −12	−12 ÷ 3 = −4
3 × −4 = −12	−12 ÷ −4 = 3
−3 × −4 = 12	12 ÷ −3 = −4
−3 × −4 = 12	12 ÷ −4 = −3

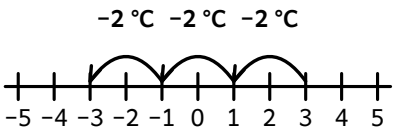
Each times fact gives two divide facts. The same sign rule works for both.

SEE IT THREE WAYS

With things

At 3 °C the temperature starts to fall. It falls 2 degrees each hour for 3 hours. What is the total change, and where does it end?

As a picture



In symbols

3 × −2 = −6. The change is −6, so the temperature ends at 3 + −6 = −3.

TRY IT

Carry on a pattern

You need: a sheet of paper, a pencil.

1. Write 3 × −2 = −6 at the top of the page.
2. Write 2 × −2 under it. Work out the answer by taking the answer above and adding 2.
3. Go on down to −3 × −2. Each time the first number goes down by 1.
4. Look at the answers. Do they go up or down?

What you will see: The answers are −6, −4, −2, 0, 2, 4, 6. They go up by 2 each time, so −3 × −2 = 6.

If you cannot try it, this is what happens.

sum	answer
3 × −2	−6
2 × −2	−4
1 × −2	−2
0 × −2	0
−1 × −2	2
−2 × −2	4
−3 × −2	6

A WRONG IDEA MANY PEOPLE HAVE

~~**A negative times a negative is a negative, so −3 × −4 = −12.**~~

Why it feels right: Two minus signs feel twice as negative. And 3 × −4 really is −12, so the minus sign seems to win every time.

The test that breaks it: Look at the pattern table. 0 × −4 is 0. The next row has one less lot of −4, and the answer goes up by 4, not down. So −1 × −4 is 4.

What is true: Two numbers with the same sign give a positive answer. So −3 × −4 = 12.

KEY WORDS

rise
how many degrees the temperature goes up.

product
the answer when you multiply numbers.

BACK TO YOUR GUESS

The answer is B: The rise is 8 degrees. It goes from −3 up to 0, and then on up to 5.

First the temperature climbs 3 degrees to reach 0. Then it climbs 5 more. 3 + 5 = 8 degrees. You count across zero. You do not take 3 away from 5.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. At night it is −6 °C. By noon it is 4 °C. How many degrees did it rise?

   A 2 degrees

   B 4 degrees

   C 10 degrees

   Answer at the back of the book.

2. Work out −8 + 5.

   A −13

   B −3

   C 3

   Answer at the back of the book.

3. Work out 3 − 9.

   A −6

   B 6

   C 12

   Answer at the back of the book.

4. What is −5 × −2?

   A −10

   B −7

   C 10

   Answer at the back of the book.

5. Work out −24 ÷ 6.

   A −18

   B −4

   C 4

   Answer at the back of the book.

6. At night it is −2 °C. By noon it is 7 °C. How many degrees did it rise?

   Answer at the back of the book.

7. Work out −7 × 3.

   Answer at the back of the book.

8. **From an earlier chapter**

Which number is the smallest: 4, −2, −7 or 0?

A 0

B −7

C −2

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Hop right to add a positive number and left to take one away. Adding a negative moves you left. Same signs multiply to a positive answer. Different signs give a negative answer. Dividing follows the same rule.

Draw: A number line from −5 to 5 with a hop from −4 to 3 labelled +7. Under it write −4 + 7 = 3.

M3.1.3 CHAPTER 3

Multiples and factors

Which numbers share out 36 exactly, with nothing left over?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** The factors of 36 are 36, 72, 108 and so on.
- ☐ **B** The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.
- ☐ **C** The factors of 36 are 2, 3, 4, 6, 9 and 18 only.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.6. If you cannot yet write a times fact and a divide fact together, read this first.
- Level 2, Times tables and sharing, M2.3.5. If the 6, 7 and 9 tables are not firm yet, practise them first.

BY THE END YOU CAN

- List the factors of 36 in pairs
- Find common multiples of 4 and 6

Joseph has 36 stones. He wants to lay them out in equal rows, with none left over. Which row lengths can he use?

A **factor** of 36 is a whole number that divides into 36 exactly. 4 is a factor of 36, because 36 ÷ 4 = 9. Two factors that multiply to 36, such as 4 and 9, are a **factor pair**.

DRAW IT FIRST

Lay out 36 tiles in equal rows

1 × 36

1 Make 1 row of 36. This shows 1 × 36 = 36. The pair is 1 and 36.

1 × 36

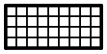
2 × 18 

2 Try 2 rows. 36 ÷ 2 = 18, so 2 rows of 18 work. The pair is 2 and 18.

1 × 36 

2 × 18 

3 × 12 

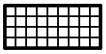
4 × 9 

3 Try 3 rows and 4 rows. 3 rows of 12 and 4 rows of 9 both work.

1 × 36 

2 × 18 

3 × 12 

4 × 9 

6 × 6 

4 Try 6 rows. 6 rows of 6 work. This pair is 6 and 6, so we write 6 only once.

What about 5 rows? 36 ÷ 5 leaves 1 over, so 5 is not a factor. After 6 the pairs repeat in reverse, 9 and 4, then 12 and 3. So you can stop at 6.

WORKED EXAMPLE

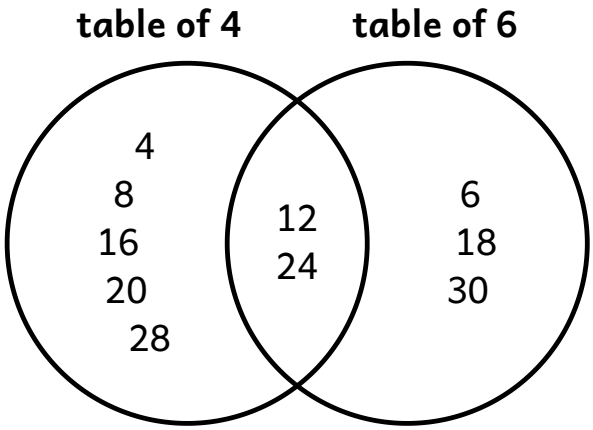
List the factors of 36.

1. Try the numbers in turn. 36 ÷ 1 = 36, 36 ÷ 2 = 18, 36 ÷ 3 = 12 and 36 ÷ 4 = 9.
2. Then 36 ÷ 6 = 6, so 6 pairs with itself. After that the pairs only repeat, so stop at 6.
3. Write every number from the pairs in order, with 6 once. The factors are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

Answer: 1, 2, 3, 4, 6, 9, 12, 18 and 36

A **multiple** of 4 is a number in the 4 times table. The first ones are 4, 8, 12, 16, 20, 24 and 28. Multiples go on for ever. A factor is never bigger than the number.

The multiples of 6 are 6, 12, 18, 24, 30 and so on. Some numbers are in both lists. 12 and 24 are in both. We call each one a **common multiple** of 4 and 6.



Multiples of 4 and of 6, up to 30. The numbers in the middle are in both times tables.

table of 4	table of 6
4	6
8	12
12	18
16	24
20	30
24	36

The first six multiples of 4 and of 6. The numbers 12 and 24 are in both columns, and 12 is the first.

WORKED EXAMPLE

Find the first three common multiples of 4 and 6.

1. Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36.
2. Multiples of 6: 6, 12, 18, 24, 30, 36.
3. The numbers in both lists are 12, 24 and 36.

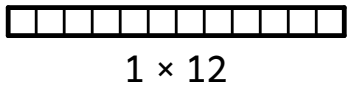
Answer: 12, 24 and 36

SEE IT THREE WAYS

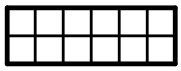
With things

Take 12 stones. Lay them out in equal rows. You can make 1 row of 12, 2 rows of 6, or 3 rows of 4.

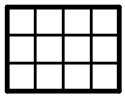
As a picture



1 × 12



2 × 6



3 × 4

In symbols

1 × 12 = 12, 2 × 6 = 12 and 3 × 4 = 12. The factors of 12 are 1, 2, 3, 4, 6 and 12.

TRY IT

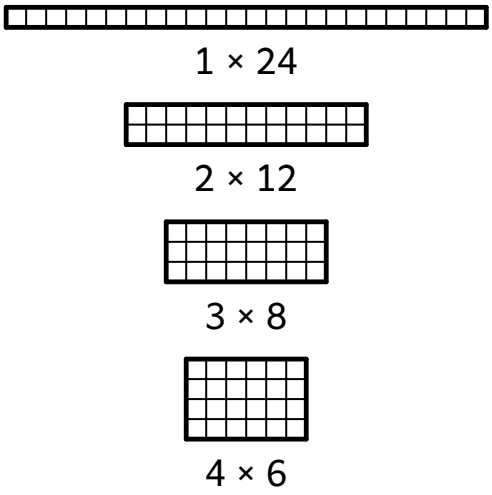
Find the factor pairs of 24

You need: 24 small stones, beans or buttons.

1. Lay out all 24 in one row.
2. Make 2 equal rows. Then try 3 rows, 4 rows, 5 rows, 6 rows and 7 rows.
3. Each time, see if any stones are left over. Write down the row counts that use every stone.

What you will see: Rows of 2, 3, 4 and 6 work, and so do 1 and 24. Rows of 5 and 7 leave stones over. The factor pairs are 1 and 24, 2 and 12, 3 and 8, 4 and 6.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~The factors of 4 are 4, 8, 12, 16 and so on.~~

Why it feels right: Factors and multiples are both about times tables, and the two words sound alike.

The test that breaks it: Try 4 ÷ 8. You cannot make groups of 8 from 4 stones. But 4 ÷ 1 = 4, 4 ÷ 2 = 2 and 4 ÷ 4 = 1, with nothing left.

What is true: The factors of 4 are 1, 2 and 4. A factor is never bigger than the number. The list 4, 8, 12 holds multiples of 4, and a multiple is never smaller.

KEY WORDS

factor
a whole number that divides into another number exactly.

factor pair
two factors that multiply to make the number, such as 4 and 9 for 36.

multiple
a number in the times table of another number.

common multiple
a number that is a multiple of two or more numbers.

BACK TO YOUR GUESS

The answer is B: The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

The numbers that share out 36 exactly are 1, 2, 3, 4, 6, 9, 12, 18 and 36. Do not leave out 1 and 36 themselves. The list 36, 72, 108 shows multiples of 36, not factors.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which list shows all the factors of 20?

   A 2, 4, 5 and 10

   B 20, 40, 60, 80 and 100

   C 1, 2, 4, 5, 10 and 20

Answer at the back of the book.

2. Which of these numbers is a factor of 42? The picture shows 42 tiles in one row.



42 tiles

A 9

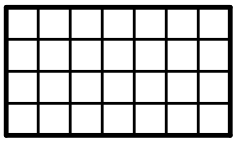
B 15

C 6

D 8

Answer at the back of the book.

3. Which list shows the factor pairs of 28?



4 × 7 = 28

A 1 × 28, 2 × 14 and 4 × 7

B 2 × 14 and 4 × 7

C 1 × 28, 2 × 14, 3 × 9 and 4 × 7

Answer at the back of the book.

4. Which list shows the first five multiples of 7?

A 14, 21, 28, 35 and 42

B 7, 17, 27, 37 and 47

C 7, 14, 21, 28 and 35

Answer at the back of the book.

5. Which of these numbers is a common multiple of 3 and 5?

A 25

B 30

C 20

Answer at the back of the book.

6. Write the first five multiples of 6.

   Answer at the back of the book.

7. How many factors does 24 have?

   Answer at the back of the book.

8. **From an earlier chapter**

   Work out -6×-7.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Factors divide a number exactly and come in pairs: 36 = 4 × 9. Multiples are the times table: 4, 8, 12. Common multiples are in two lists, like 12 and 24 for 4 and 6.

Draw: Draw four rectangles of squares for the factor pairs of 24. Write 1 × 24, 2 × 12, 3 × 8 and 4 × 6 beside them.

M3.1.4 CHAPTER 4

Quick tests for dividing

How can you tell if a number divides exactly, without doing the division?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** A number is divisible by 5 if its first digit is 5 or 0.
- ☐ **B** A number is divisible by 5 if its last digit is 0 or 5.
- ☐ **C** A number is divisible by 5 if its digits add up to 5.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.2. If the last digits of the 2, 5 and 10 tables are new, read this first.

BY THE END YOU CAN

- Use the last digit to test for 2, 5 and 10
- Use the digit sum to test for 3 and 9

Ana has 4 536 beans. She wants to share them equally between 9 people. Must she do the long division to find out if it works?

A number is **divisible** by 5 if it divides by 5 exactly, with nothing left over. For some numbers there is a quick test. You only look at the digits.

The multiples of 10 are 10, 20, 30, 40. They all end in 0. A number ends in 0 only if it is a multiple of 10. So the test for 10 is the last digit.

The tests for 2 and 5 use the last digit too. A number is divisible by 2 if its last digit is 0, 2, 4, 6 or 8. It is divisible by 5 if its last digit is 0 or 5.

table	last digits
2	0 2 4 6 8
5	0 5
10	0

The last digits that a multiple of 2, 5 or 10 can have.

Why only the last digit? Because 10 = 2 × 5. Every ten in a number divides by 2, by 5 and by 10. So only the ones digit can leave something over.

number	by 2	by 5	by 10
340	Yes	Yes	Yes
725	No	Yes	No
418	Yes	No	No

Each number is tested by its last digit. 340 ends in 0, so it passes all three tests.

WORKED EXAMPLE

Is 2 460 divisible by 2, by 5 and by 10?

1. Look at the last digit. It is 0.
2. A number ending in 0 is divisible by 2, by 5 and by 10.
3. Check: 2 460 ÷ 10 = 246, and 246 is a whole number.

Answer: Yes to all three, because the last digit is 0.

For 3 and 9 you need every digit. Add the digits up. The answer is the **digit sum**. If the digit sum is in the 3 times table, the number is divisible by 3. If it is in the 9 times table, the number is divisible by 9.

DRAW IT FIRST

Find the digit sum of 4 536

thousands	hundreds	tens	ones
4	5	3	6

1 Write 4 536 in a place value chart. There is one digit in each place.

thousands	hundreds	tens	ones
4	5	3	6

4 + 5 + 3 + 6 = 18

2 Add the digits: 4 + 5 + 3 + 6 = 18.

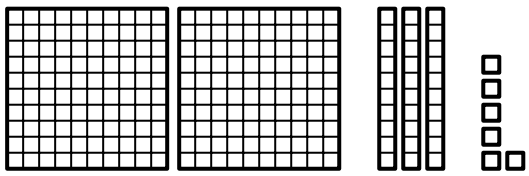
thousands	hundreds	tens	ones
4	5	3	6

4 + 5 + 3 + 6 = 18

18 = 2 × 9
divisible by 9 and by 3

3 The digit sum 18 is in the 9 times table. So 4 536 is divisible by 9, and by 3 as well.

Why does this work? Take 236. It is 2 hundreds, 3 tens and 6 ones. A hundred is 99 + 1. A ten is 9 + 1.



236

Every flat is 99 and one more. Every rod is 9 and one more.

So 236 is 2 × 99 + 3 × 9, and then 2 + 3 + 6 more. The first part is always in the 9 times table. It is in the 3 times table too. Only the digit sum 2 + 3 + 6 = 11 can leave something over.

11 ÷ 9 leaves 2, and 236 ÷ 9 leaves 2 as well. 11 ÷ 3 leaves 2, and 236 ÷ 3 leaves 2 too. The digit sum leaves the same remainder as the whole number.

WORKED EXAMPLE

Is 7 428 divisible by 3?

1. Add the digits: 7 + 4 + 2 + 8 = 21.
2. Is 21 in the 3 times table? Yes, 3 × 7 = 21.
3. So 7 428 is divisible by 3.

Answer: Yes, because the digit sum is 21.

WORKED EXAMPLE

Is 7 428 divisible by 9?

1. Add the digits: 7 + 4 + 2 + 8 = 21.
2. The 9 times table has 9 × 2 = 18 and 9 × 3 = 27.
3. 21 is between 18 and 27, so 21 is not in the 9 times table. 21 ÷ 9 = 2 r 3.
4. So 7 428 is not divisible by 9.

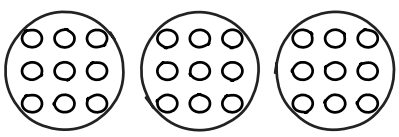
Answer: No. The digit sum is 21, and that is not divisible by nine.

SEE IT THREE WAYS

With things

Take 27 stones. Make groups of 9. You get 3 groups and no stones are left over.

As a picture



In symbols

27: 2 + 7 = 9. The digit sum is 9, so 27 is divisible by 9. 27 ÷ 9 = 3.

TRY IT

Add the digits of the 9 times table

You need: a sheet of paper, a pencil.

1. Write the 9 times table from 9 to 81: 9, 18, 27 and so on.
2. Next to each number, write its digit sum.
3. Look down the digit sums. What do you see?

What you will see: Every digit sum is 9. A number in the 9 times table has a digit sum in the 9 times table.

If you cannot try it, this is what happens.

number	digit sum
9	9
18	9
27	9
36	9
45	9
54	9
63	9
72	9
81	9

A WRONG IDEA MANY PEOPLE HAVE

~~A number that ends in 3 is divisible by 3.~~

Why it feels right: The tests for 2, 5 and 10 look only at the last digit. So it seems the same should work for 3.

The test that breaks it: Try 13. 13 ÷ 3 leaves 1. Now try 33. 33 ÷ 3 = 11 with nothing left. Both end in 3, but only one divides.

What is true: For 3 and 9 every digit counts. Add them up and test the digit sum. 1 + 3 = 4 is not in the 3 times table, but 3 + 3 = 6 is.

KEY WORDS

divisible
can be divided exactly, with nothing left over.

digit sum
the answer when you add up all the digits of a number.

BACK TO YOUR GUESS

The answer is B: A number is divisible by 5 if its last digit is 0 or 5.

Only the last digit matters for 5. A number is divisible by 5 when it ends in 0 or 5. The digit sum is for a different test, the one for 3 and 9.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which of these numbers is divisible by 5?

   A 1 115

   B 2 004

   C 1 062

   Answer at the back of the book.

2. Which of these numbers is divisible by 10?

   A 3 804

   B 3 840

   C 3 845

   Answer at the back of the book.

3. Which of these numbers is divisible by 3? Use the digit sum.

   A 5 817

   B 5 813

   C 5 810

   Answer at the back of the book.

4. Which of these numbers is divisible by 9? Use the digit sum.

   A 6 726

   B 6 703

   C 6 723

   Answer at the back of the book.

5. Which of these numbers is divisible by both 2 and 3?

   A 4 533

   B 4 536

   C 4 538

   Answer at the back of the book.

6. What is the digit sum of 5 817?

   Answer at the back of the book.

7. Amina writes 52, then one more digit on the end. Which last digit makes her three digit number divisible by 9?

   Answer at the back of the book.

8. **From an earlier chapter**

   List all the factors of 18.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Ends in 0, 2, 4, 6 or 8: divisible by 2. Ends in 0 or 5: by 5. Ends in 0: by 10. Digit sum in the 3 or 9 table: the number is too.

Draw: A number such as 4 536 in four boxes, with its digits added up underneath. Write 4 + 5 + 3 + 6 = 18.

M3.1.5 CHAPTER 5

Prime numbers

> **Why can 7 stones go in only one row, but 6 stones can make two rows?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** A prime number is any odd number.
- ☐ **B** A prime number has exactly two factors: 1 and itself.
- ☐ **C** A prime number is a number that you cannot divide by 2.

IF THIS FEELS HARD, READ THIS FIRST

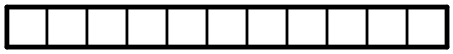
- Level 2, Times tables and sharing, M2.3.5. Finding primes uses the 7 times table. Practise it here first.

BY THE END YOU CAN

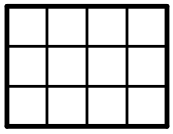
- Say if a number up to 100 is prime
- Find all primes up to 100 by crossing out multiples

Mei has 7 stones. She tries to lay them out in equal rows. She can make 1 row of 7. She cannot make 2 rows, or 3, or 4. Her friend has 6 stones and can make 2 rows of 3.

A **prime number** has exactly two factors: 1 and itself. 7 is prime, because its only factors are 1 and 7. A **composite number** has more than two factors. 6 is composite. Its factors are 1, 2, 3 and 6.



11 stones: 1 × 11 only

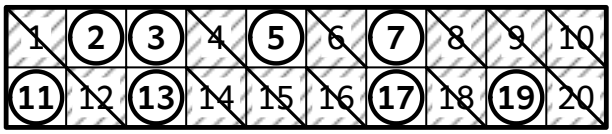


12 stones: 3 × 4

11 is prime: it cannot be laid out in more than one row of equal length. 12 is composite.

1 is neither prime nor composite. It has only one factor, which is itself. 2 is prime. It is the only even prime. Every other even number divides by 2, so it has at least three factors.

The primes below 20 are 2, 3, 5, 7, 11, 13, 17 and 19. The other numbers from 4 to 18 are composite.



The primes below 20 are ringed. Every other number is crossed out.

To find every prime up to 100, cross out numbers in a hundred square. This method is called a **sieve**. Like a kitchen sieve, it keeps the numbers you want and lets the others fall out.

In the pictures, a crossed out number has a line through its cell and is shaded. A prime has a ring round it and bold digits.

DRAW IT FIRST

Find the primes up to 100 with *a* sieve

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1 Cross out 1, which is not prime. Ring 2, the first prime.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2 Cross out every multiple of 2 after 2. Ring 3. Cross out every multiple of 3 after 3.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

3 Do the same for 5 and 7. Every number left is a prime. Ring them all.

Why stop at 7? The next prime is 11, and 11 × 11 = 121 is more than 100. A composite number up to 100 has a factor of 10 or less, other than 1. That factor is 2, 3, 5 or 7, or divides by one of them. So the sieve has crossed it out.

You can also test one number from 2 to 100 without a sieve. Divide it by 2, 3, 5 and 7. If none of them divides it exactly, it is prime. This test only works up to 100.

divide by	remainder
2	1
3	1
5	1
7	0

The remainders when 91 is divided by 2, 3, 5 and 7.

WORKED EXAMPLE

Is 91 prime?

1. Test 2, 3, 5 and 7. 91 is odd, so 2 fails. For 3, the digit sum is 9 + 1 = 10.
2. 10 is not in the 3 times table, so 3 fails. 91 does not end in 0 or 5, so 5 fails.
3. Now try 7. 91 ÷ 7 = 13, with nothing left over. So 7 is a factor and 91 is not prime.

Answer: No, 91 = 7 × 13.

WORKED EXAMPLE

Is 59 prime?

1. 59 is odd, so 2 fails. For 3, the digit sum is 5 + 9 = 14.
2. 14 is not in the 3 times table, so 3 fails. 59 ends in 9, so 5 fails.
3. For 7, look at 7 × 8 = 56 and 7 × 9 = 63.
4. Neither is 59, so 7 fails. No test finds a factor, so 59 is prime.

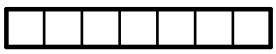
Answer: Yes, 59 is prime.

SEE IT THREE WAYS

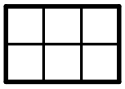
With things

Take 7 stones. Try to make equal rows. Only one row of 7 works. Now take 6 stones. You can make 2 rows of 3, or 3 rows of 2.

As a picture



7 stones: 1 × 7 only



6 stones: 2 × 3

In symbols

7 has two factors, 1 and 7, so it is prime. 6 has four factors, 1, 2, 3 and 6, so it is composite.

TRY IT

Make a sieve to 50

You need: a sheet of paper, a pencil, a ruler or a straight edge.

1. Write the numbers 1 to 50 in rows of 10.
2. Cross out 1. Ring 2. Cross out every second number after 2.
3. Ring 3. Cross out every third number after 3. Do the same for 5 and 7.
4. Ring every number that is left. Count the rings.

What you will see: There are 15 rings. The first ten primes are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29. After them come 31, 37, 41, 43 and 47.

If you cannot try it, this is what happens.

1	(2)	(3)	4	(5)	6	(7)	8	9	10
(11)	12	(13)	14	15	16	(17)	18	(19)	20
21	22	(23)	24	25	26	27	28	(29)	30
(31)	32	33	34	35	36	(37)	38	39	40
(41)	42	(43)	44	45	46	(47)	48	49	50

A WRONG IDEA MANY PEOPLE HAVE

~~Every odd number is prime.~~

Why it feels right: All the even numbers after 2 are composite. So the odd numbers look like the ones that are left.

The test that breaks it: Try 9. 9 = 3 × 3, so it has three factors. Try 15. 15 = 3 × 5. Try 21 and 25 too.

What is true: Odd numbers can be composite. 9, 15, 21, 25 and 27 are all odd and composite. Only the numbers with exactly two factors are prime.

KEY WORDS

prime number
a number with exactly two factors, 1 and itself.

composite number
a number with more than two factors.

sieve
a way to find primes by crossing out multiples.

BACK TO YOUR GUESS

The answer is B: A prime number has exactly two factors: 1 and itself.

A prime number has exactly two factors, 1 and itself. 9 is odd but 3 divides it, so 9 is not prime. And 2 is prime, even though it is in the 2 times table.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Is 51 prime? Say why.

   A Yes, 51 is odd.

   B Yes, because 2, 5 and 7 do not divide it.

   C No, $51 = 3 \times 17$.

   Answer at the back of the book.

2. Which of these numbers is prime?

   A 59

   B 65

   C 57

   D 63

   Answer at the back of the book.

3. Which of these statements is true?

   A The number 2 is the only even prime.

   B Every even number is composite.

   C The number 1 is the smallest prime.

   Answer at the back of the book.

4. The picture shows a sieve to 30 after 2 and 3. Which numbers do you cross out next?

1	**2**	**3**	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

   A The multiples of 4

   B The multiples of 6

   C The multiples of 5

   Answer at the back of the book.

5. Why do we cross out only multiples of 2, 3, 5 and 7 up to 100?

   A Because 7 is the biggest prime with one digit.

   B The next prime is 11, and $11 \times 11 = 121$ is more than 100.

   C There are no primes after 7.

   Answer at the back of the book.

6. What is the smallest prime factor of 91?

   Answer at the back of the book.

7. How many primes are there from 1 to 30?

   Answer at the back of the book.

8. Look at the last picture of the sieve in "Draw it first". Count the ringed numbers. How many primes are there up to 100?

   Answer at the back of the book.

9. **From an earlier chapter**

   Is 8 124 divisible by 3? Use the digit sum.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A prime has exactly two factors, 1 and itself. 1 is not prime. 2 is the only even prime. There are 25 primes up to 100. For a number from 2 to 100, test with 2, 3, 5 and 7.

Draw: A square of the numbers 1 to 30 in rows of 10. Cross out the composite numbers and ring the primes.

M3.1.6 CHAPTER 6

Prime factors, common factors and multiples

Can you build every whole number from prime numbers by multiplying?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 60 = 6 × 10, so the prime factors of 60 are 6 and 10.
- ☐ **B** 60 = 2 × 2 × 3 × 5, and no other list of primes multiplies to 60.
- ☐ **C** 60 = 2 × 30, so the prime factors of 60 are 2 and 30.

BY THE END YOU CAN

- Write 60 as a product of primes using a factor tree
- Find the highest common factor of 24 and 36
- Find the lowest common multiple of 6 and 8

A prime number cannot be split into smaller factors. Every other whole number bigger than 1 can be split. The pieces can be split again, until only primes are left. Those primes are the **prime factors**. So the answer to the question is yes, for every whole number bigger than 1.

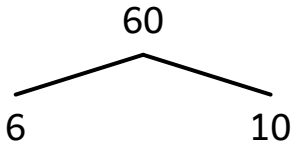
A **factor tree** shows the splitting. Write the number at the top. Split it into a factor pair. Split each number that is not prime. Stop when every end number is prime.

DRAW IT FIRST

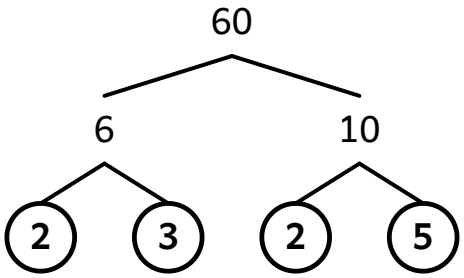
Build a factor tree for 60

60

1 Write 60 at the top.



2 Split 60 into a factor pair, such as 6 and 10. Neither is prime, so go on.



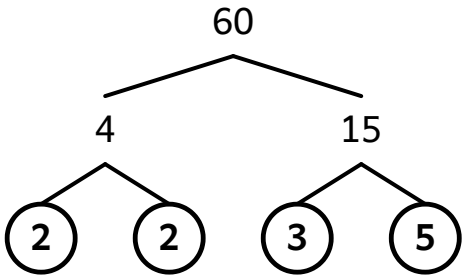
3 Split 6 into 2 and 3, and 10 into 2 and 5. Every end number is prime, so stop. They are ringed.

WORKED EXAMPLE

Write 60 as a product of primes.

1. Split 60 into 6 × 10. Check: 6 × 10 = 60.
2. Split 6 into 2 × 3, and 10 into 2 × 5.
3. The end numbers are 2, 3, 2 and 5. Write them with × between them, smallest first.

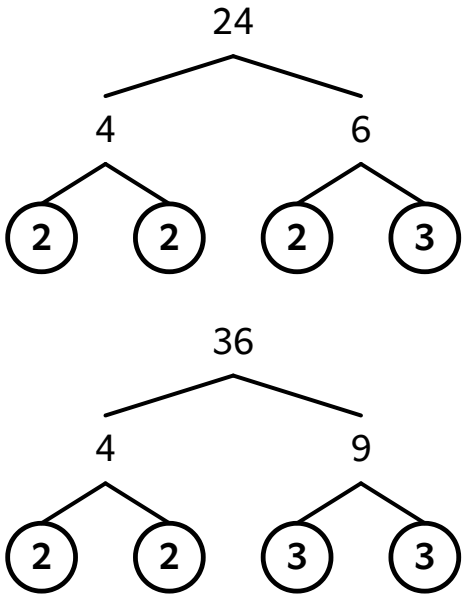
Answer: 60 = 2 × 2 × 3 × 5



A different start, 4 × 15. The end numbers are the same primes: 2, 2, 3 and 5.

Any way of splitting a number ends with the same primes. There is only one list of prime factors for each number. That is why we can use it to compare numbers.

The **highest common factor** (HCF) of two numbers is the biggest number that is a factor of both. Take each prime as many times as both numbers have it, and multiply. If they share no prime, the HCF is 1.



24 = 2 × 2 × 2 × 3 and 36 = 2 × 2 × 3 × 3. Both have 2, 2 and 3.

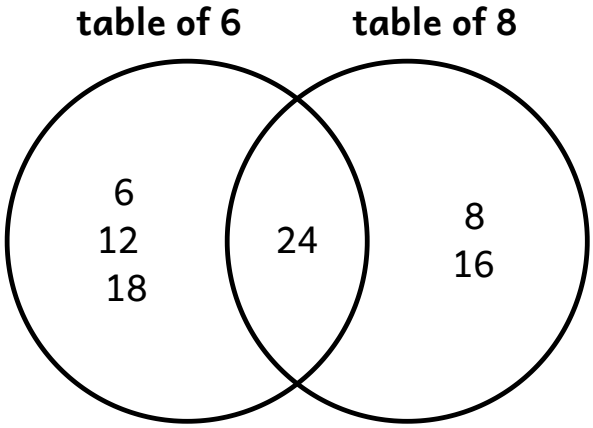
WORKED EXAMPLE

Find the highest common factor of 24 and 36.

1. The primes of 24 are 2 × 2 × 2 × 3. Check: 2 × 2 × 2 × 3 = 24.
2. The primes of 36 are 2 × 2 × 3 × 3. Check: 2 × 2 × 3 × 3 = 36.
3. The shared primes are 2, 2 and 3. 2 × 2 × 3 = 12.

Answer: 12

The **lowest common multiple** (LCM) of two numbers is the smallest number that is a multiple of both. List the multiples of each number and find the first one they share.



The multiples of 6 and of 8, up to 24. Only one number is in both so far.

WORKED EXAMPLE

Find the lowest common multiple of 6 and 8.

1. Multiples of 6: 6, 12, 18, 24, 30.
2. Multiples of 8: 8, 16, 24, 32.
3. The first number that is in both lists is 24.
4. So 24 is in both tables. Check: 24 ÷ 6 = 4 and 24 ÷ 8 = 3.

Answer: 24

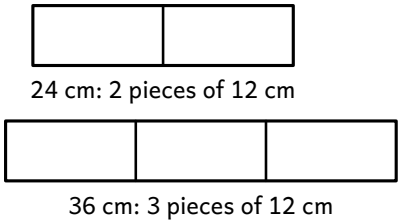
You can also use primes. 6 = 2 × 3 and 8 = 2 × 2 × 2. For each prime, take the most times it appears in either number. That gives 2 × 2 × 2 × 3 = 24.

SEE IT THREE WAYS

With things

Kofi has two strips of cloth, 24 cm and 36 cm long. He cuts both into equal pieces with none left over. What is the longest piece he can use?

As a picture



In symbols

24 = 2 × 12 and 36 = 3 × 12. The highest common factor of 24 and 36 is 12.

TRY IT

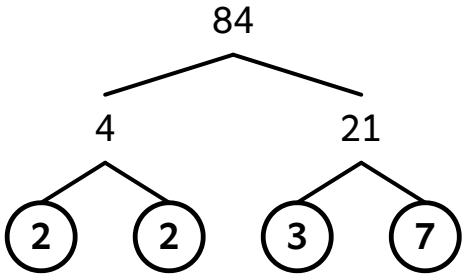
Make a factor tree for 84

You need: a sheet of paper, a pencil.

1. Write 84 at the top.
2. Split it into any factor pair, such as 4 × 21. Split each number that is not prime.
3. Stop when every end number is prime. Write the primes with × between them.
4. Start again with a different pair, such as 6 × 14. Do you get the same primes?

What you will see: Both trees end with 2, 2, 3 and 7. 2 × 2 × 3 × 7 = 84.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~The lowest common multiple of 6 and 8 is 2, because 2 goes into both.~~

Why it feels right: The words "lowest" and "common" sound like the smallest number that the two numbers share.

The test that breaks it: Is 2 in the 6 times table? The multiples of 6 are 6, 12, 18, and so on. None is as small as 2.

What is true: A multiple is never smaller than the number. 2 is a common factor of 6 and 8. The lowest common multiple is 24.

KEY WORDS

prime factors
the factors of a number that are prime numbers.

factor tree
a diagram that splits a number into smaller factors until only primes are left.

highest common factor
the biggest number that is a factor of two numbers (HCF).

lowest common multiple
the smallest number that is a multiple of two numbers (LCM).

BACK TO YOUR GUESS

The answer is B: 60 = 2 × 2 × 3 × 5, and no other list of primes multiplies to 60.

Prime factors must all be prime. 6, 10 and 30 are not prime, so keep splitting. You end with 60 = 2 × 2 × 3 × 5. Whichever way you split, the primes are the same.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which of these is 36 written as a product of primes?

   A 4 × 9

   B 2 × 18

   C 2 × 2 × 3 × 3

   Answer at the back of the book.

2. Which of these is 90 written as a product of primes?

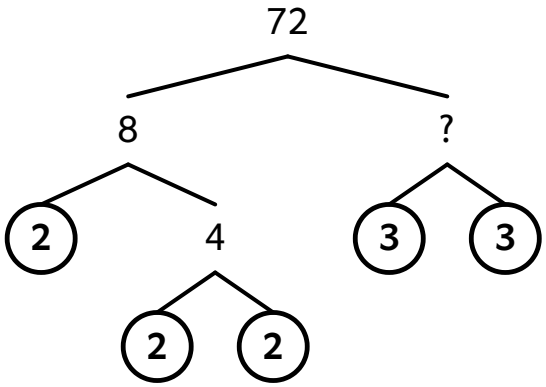
   A 2 × 5 × 9

   B 2 × 3 × 3 × 5

   C 9 × 10

   Answer at the back of the book.

3. Look at the factor tree. What number is hidden by the question mark?



   A 9

   B 64

   C 80

   Answer at the back of the book.

4. Find the highest common factor of 18 and 30.

   A 3

   B 90

   C 6

   Answer at the back of the book.

5. Find the lowest common multiple of 4 and 10.

   A 40

   B 20

   C 2

   Answer at the back of the book.

6. Find the highest common factor of 16 and 40.

   Answer at the back of the book.

7. Find the lowest common multiple of 12 and 18.

   Answer at the back of the book.

8. **From an earlier chapter**

   Is 87 prime? Say why.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Split a number into a factor pair, and keep splitting until every end is prime. 60 = 2 × 2 × 3 × 5. The HCF is the biggest shared factor. The LCM is the smallest shared multiple.

Draw: Draw a factor tree for 60. Put 6 and 10 below it, then 2, 3, 2 and 5. Ring each prime.

M3.1.7 CHAPTER 7

Squares, cubes and roots

What does the small raised 2 in 5^2 mean?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 5^2 means 5 × 2, which is 10.
- ☐ **B** 5^2 means 5 × 5, which is 25.
- ☐ **C** 5^2 means 5 + 5, which is 10.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Patterns and rules, M2.8.6. If you have not seen square numbers drawn as dots, read this first.

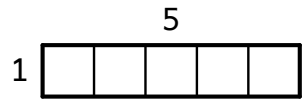
BY THE END YOU CAN

- Write 2 × 2 × 2 as 2^3, and 5 squared as 5^2
- Find the square root of a square number up to 144
- Find the cube root of 8, 27 and 64

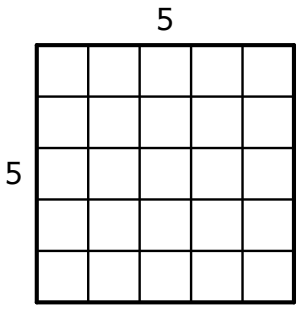
Ana lays floor tiles in a square, with 5 tiles along each side. How many tiles does she need?

DRAW IT FIRST

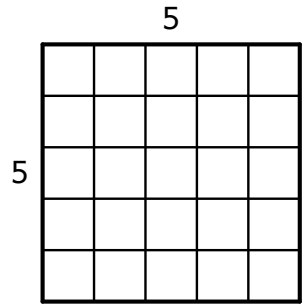
Build *a* square of 5 tiles by 5 tiles



1 Lay one row of 5 tiles.



2 Lay 5 rows of 5 tiles. The tiles make *a* square.



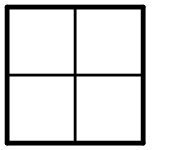
5 × 5 = 25

3 Count the tiles: 5 × 5 = 25. We write 5 × 5 as 5^2, and say "5 squared".

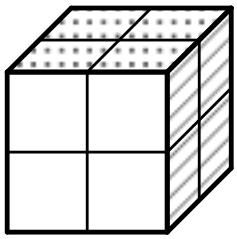
Multiplying *a* number by itself gives *a* **square number**. 25 is *a* square number, because it is 5 × 5. We call *a* number written like 5^2 *a* **power**.

In 5^2, the small raised 2 is the **index**. It tells you how many 5s to multiply together. Some books call it the exponent.

Now think of *a* block made of small cubes. A **cube number** comes from multiplying three equal whole numbers. For example, 2 × 2 × 2 = 8. We write it 2^3 and say "2 cubed".



one layer: 4



two layers: 8

One layer has 2 × 2 = 4 small cubes. Two layers have 4 + 4 = 8, which is 2 × 2 × 2.

WORKED EXAMPLE

Write 3 × 3 × 3 × 3 as a power, and work out its value.

1. Count the 3s. There are four of them. So the index is 4.
2. Write 3^4.
3. 3 × 3 = 9, then 9 × 3 = 27, then 27 × 3 = 81.

Answer: $3^4 = 81$

Here are the square numbers up to 144. Learn them. They will turn up again and again.

n	n^2	n	n^2
1	1	7	49
2	4	8	64
3	9	9	81
4	16	10	100
5	25	11	121
6	36	12	144

The squares of 1 to 12.

And here are the first cube numbers.

n	n^3
1	1
2	8
3	27
4	64
5	125

The cubes of 1 to 5.

Other powers work the same way. $2^4 = 2 \times 2 \times 2 \times 2 = 16$. Squares and cubes are the powers with index 2 and index 3.

Now go backwards. $5^2 = 25$, so the **square root** of 25 is 5. We write $\sqrt{25} = 5$. The square root is the positive number that multiplies by itself to give the number. Also, $-5 \times -5 = 25$, but $\sqrt{25}$ always means 5.

In the same way, $2^3 = 8$, so the **cube root** of 8 is 2. The cube root of a number is the number that gives it when it is cubed.

WORKED EXAMPLE

Find the square root of 144.

1. Look for a number that multiplies by itself to make 144.
2. $10 \times 10 = 100$ is too small. $11 \times 11 = 121$ is still too small.
3. $12 \times 12 = 144$. So $\sqrt{144} = 12$.

Answer: $\sqrt{144} = 12$

WORKED EXAMPLE

Find the cube roots of 8, 27 and 64.

1. $2 \times 2 \times 2 = 8$, so the cube root of 8 is 2.
2. $3 \times 3 \times 3 = 27$, so the cube root of 27 is 3.
3. $4 \times 4 \times 4 = 64$, so the cube root of 64 is 4.
4. So the cube roots of 8, 27 and 64 are 2, 3 and 4.

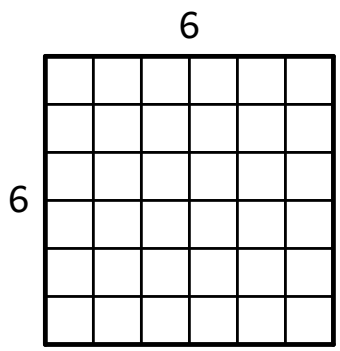
Answer: 2, 3 and 4

SEE IT THREE WAYS

With things

A square floor has 36 tiles in equal rows. How many tiles are along one side?

As a picture



In symbols

$6 \times 6 = 36$, so $6^2 = 36$. The square root of 36 is 6, and we write $\sqrt{36} = 6$.

TRY IT

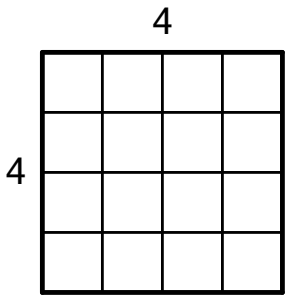
Make a square from stones

You need: 20 small stones, beans or buttons.

1. Count out 16 stones. Try to lay them in a square, with the same number in each row and column.
2. Put them back and count out 12 stones. Try again.
3. Try 9 stones, then 20 stones. Which numbers make a square?

What you will see: The 16 stones make a square of 4 rows of 4. The 9 stones make 3 rows of 3. The 12 and 20 stones do not. Only square numbers make a square.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~**5^2 means 5 × 2, so 5^2 = 10.**~~

Why it feels right: The small 2 looks like a number to multiply by, and 5 and 2 are both there.

The test that breaks it: Make a square of tiles, 5 along each side. Count them. There are 25 tiles, not 10. The same goes for 2^3: a 2 by 2 by 2 block has 8 cubes, not 6.

What is true: The index is how many copies of the number you multiply together. $5^2 = 5 \times 5 = 25$ and $2^3 = 2 \times 2 \times 2 = 8$.

KEY WORDS

square number
a number made by multiplying *a* whole number by itself, such as 25.

cube number
a number made by multiplying three equal whole numbers, such as $2 \times 2 \times 2 = 8$.

power
a number written with an index, such as 5^2 or 2^3.

index
the small raised number. It says how many of the number to multiply together.

square root
the positive number that multiplies by itself to give *a* number. $\sqrt{25} = 5$.

cube root
the number that gives *a* cube number when it is cubed. The cube root of 8 is 2.

BACK TO YOUR GUESS

The answer is B: 5^2 means 5×5, which is 25.

5^2 means 5×5, which is 25. The small number tells you how many 5s to multiply together. It does not multiply the 5 by 2.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Write $2 \times 2 \times 2 \times 2 \times 2$ as *a* power, and work out its value.

   A $5^2 = 25$

   B $2 \times 5 = 10$

   C $2^5 = 32$

   Answer at the back of the book.

2. What is 7^2?

A 9

B 49

C 14

Answer at the back of the book.

3. What is 4^3?

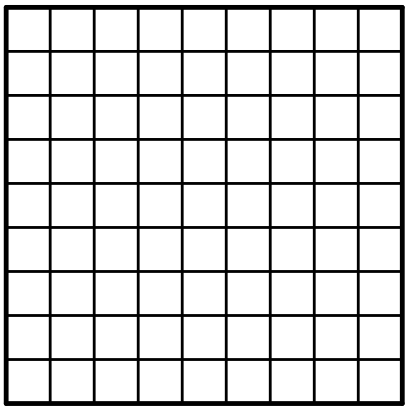
A 64

B 12

C 16

Answer at the back of the book.

4. The picture shows 81 tiles in a square. What is the square root of 81?



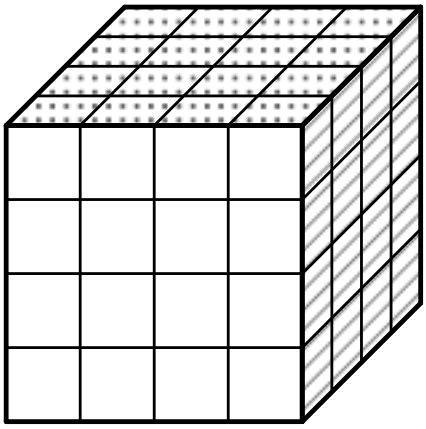
A 27

B 3

C 9

Answer at the back of the book.

5. The picture shows 64 small cubes in one big cube. What is the cube root of 64?



A 32

B 4

C 8

Answer at the back of the book.

6. Work out 9^2.

Answer at the back of the book.

7. Find the square root of 100.

Answer at the back of the book.

8. **From an earlier chapter**

Write 72 as a product of primes.

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

$5^2 = 5 \times 5 = 25$, *a* square number. $2^3 = 2 \times 2 \times 2 = 8$, *a* cube number. The square root of 25 is 5. The cube root of 8 is 2. The index counts how many of the number you multiply together.

Draw: A square of 5 by 5 small squares with 25 written under it. Next to it, draw *a* cube made of 8 small cubes and write 8.

M3.1.8 CHAPTER 8

Powers of ten

> **How many zeros are there in ten million, and is there a short way to write it?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 10^6 means 10 × 6, so it is 60.
- ☐ **B** 10^6 is 106, with the digits 10 and 6 side by side.
- ☐ **C** 10^6 is a 1 followed by six zeros, so it is 1 000 000.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Big numbers and place value, M2.1.6. If a million or a lakh is new to you, read this first.

BY THE END YOU CAN

- Write 1 000 as 10^3 and 1 000 000 as 10^6
- Say 10 000 000 as ten million or one crore
- Use powers of ten to compare very big numbers

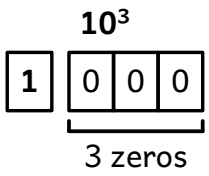
Sipho reads that a village water tank holds 100 000 litres. A pond holds 10 000 000 litres. Counting the zeros one by one is slow, and you may lose count. A power of ten is a short way to write it.

You know that 10 × 10 = 100 and 10 × 10 × 10 = 1 000. In chapter 7 you wrote 2 × 2 × 2 as 2^3. In the same way, 10 × 10 × 10 = 10^3 = 1 000, and it has three zeros.

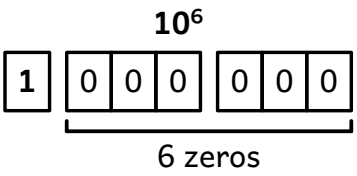
A **power of ten** is 10 with an index. The index says how many 10s are multiplied. It also tells you how many zeros follow the 1.

DRAW IT FIRST

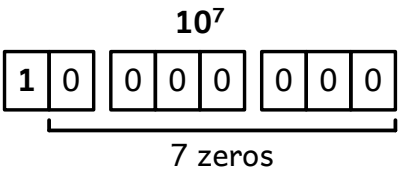
Count the zeros in three powers of ten



1 1 000 is 10^3. It is *a* 1 *and* 3 zeros, *and* the index is 3.



2 1 000 000 is 10^6. It is *a* 1 *and* 6 zeros, *and* the index is 6.



3 10 000 000 is 10^7. It is *a* 1 *and* 7 zeros, *and* the index is 7.

WORKED EXAMPLE

Write 1 000 000 *as a* power of ten.

1. Count the zeros *after* the 1. There *are* 6 of them.
2. The index is the number of zeros. So the index is 6.
3. 1 000 000 = 10^6.

Answer: 10^6

power	number
10^1	10
10^2	100
10^3	1 000
10^4	10 000
10^5	100 000
10^6	1 000 000
10^7	10 000 000

Each step down the table puts one more zero on the number.

Each time the index goes up by 1, the number gains one zero. It is 10 times as big.

Big numbers have names. In some places, 100 000 is called one **lakh** and 10 000 000 is called one **crore**. In other places 10 000 000 is ten million. All of these names mean the same numbers.

number	name
1 000	thousand
10 000	ten thousand
100 000	hundred thousand
1 000 000	million
10 000 000	ten million

One lakh is 100 000, which is a hundred thousand. One crore is 10 000 000, which is ten million.

One crore is 100 lakh, because 100 × 100 000 = 10 000 000. In powers, $10^7 = 10^2 \times 10^5$. The indices add: 2 + 5 = 7.

WORKED EXAMPLE

Say 10 000 000 in words.

1. Count the zeros after the 1. There are 7 of them, so $10\,000\,000 = 10^7$.
2. Count all the digits in 10 000 000. There are 8 digits.
3. Group the digits in threes from the right: 10, 000, 000.
4. Say it as ten million. Some places say one crore for the same number.

Answer: ten million, or one crore

To compare big numbers, look at the index first. A bigger index means more zeros, so a bigger number. 10^7 is bigger than 10^5.

How much bigger? 10^7 has two more zeros than 10^5. So 10^7 is 100 times 10^5. Two more zeros means 10 × 10 times as big.

written	number
3×10^6	3 000 000
8×10^5	800 000

The same two numbers, written with a power of ten and in full.

WORKED EXAMPLE

Which is bigger, 3×10^6 or 8×10^5?

1. $3 \times 10^6 = 3 \times 1\,000\,000 = 3\,000\,000$.
2. $8 \times 10^5 = 8 \times 100\,000 = 800\,000$.
3. Count the digits. 3 000 000 has 7 digits. 800 000 has 6 digits.
4. More digits means a bigger number, so 3 000 000 is bigger.

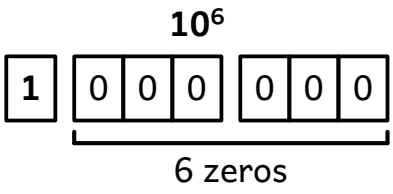
Answer: The bigger one is 3×10^6, which is 3 000 000.

SEE IT THREE WAYS

With things

A truck carries 1 000 sacks. Each sack holds 1 000 seeds. How many seeds are on the truck?

As a picture



In symbols

$10^3 \times 10^3 = 1\,000\,000 = 10^6$. The indices add: 3 + 3 = 6.

TRY IT

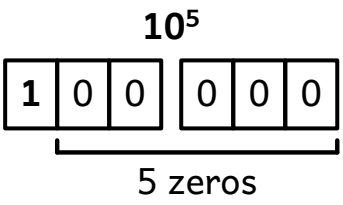
Count the zeros as you multiply

You need: a sheet of paper, a pencil.

1. Write 1 at the top of the page.
2. Under it, write 1 × 10 = 10. Then multiply the answer by 10 again, and again.
3. Each line, count the zeros in the answer and write the index beside it.
4. Stop when you reach 100 000. How many zeros has it?

What you will see: Each time you multiply by 10, one more zero arrives. 100 000 has 5 zeros, so it is 10^5. That is one lakh.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~10^3 means 10×3, so $10^3 = 30$.~~

Why it feels right: The small 3 sits next to the 10, and multiplying 10 by 3 is quick to do.

The test that breaks it: Write $10 \times 10 \times 10$. First $10 \times 10 = 100$, then $100 \times 10 = 1\,000$. There are three 10s, and the answer has three zeros. It is not 30.

What is true: The index counts how many 10s you multiply. It does not multiply 10 by that number. $10^3 = 1\,000$.

KEY WORDS

power of ten
the number 10 with an index, such as 10^3. It is a 1 followed by that many zeros.

lakh
a name used in some places for 100 000.

crore
a name used in some places for 10 000 000, which is ten million.

BACK TO YOUR GUESS

The answer is C: 10^6 is a 1 followed by six zeros, so it is 1 000 000.

10^6 means six 10s multiplied together. That is a 1 followed by six zeros, or 1 000 000. The small number counts the zeros.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. What is 10^3?

   A 30

   B 300

   C 1 000

   Answer at the back of the book.

2. Write 100 000 as a power of ten.

   A 10^4

   B 10^5

   C 10^6

   Answer at the back of the book.

3. Which of these numbers is one crore?

   A 10 000 000

   B 1 000 000

   C 100 000

   Answer at the back of the book.

4. Which is bigger, 4×10^6 or 9×10^5?

   A 9×10^5, because 9 is bigger than 4.

   B 9×10^5, because $9 \times 5 = 45$ is more than $4 \times 6 = 24$.

   C 4×10^6

   Answer at the back of the book.

5. One crore is 10 000 000 and one lakh is 100 000. How many lakh make one crore?

   A 1 000 lakh

   B 100 lakh

   C 10 lakh

   Answer at the back of the book.

6. Write 10^4 as an ordinary number.

   Answer at the back of the book.

7. Write 3×10^4 as an ordinary number.

   Answer at the back of the book.

8. **From an earlier chapter**

   Find the square root of 121.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

10^3 = 1 000, 10^6 = 1 000 000. The index counts the zeros. 10 000 000 is ten million, or one crore. 100 000 is one hundred thousand, or one lakh. A bigger index means a bigger number.

Draw: Boxes for 10^7: a 1 and seven zeros, grouped 10, 000, 000. Draw a bracket under the zeros and write 7 zeros.

Key words, A to Z

Every key word in this book, what it means, and the chapter that teaches it.

common multiple
a number that is a multiple of two or more numbers. M3.1.3

composite number
a number with more than two factors. M3.1.5

crore
a name used in some places for 10 000 000, which is ten million. M3.1.8

cube number
a number made by multiplying three equal whole numbers, such as 2 × 2 × 2 = 8. M3.1.7

cube root
the number that gives a cube number when it is cubed. The cube root of 8 is 2. M3.1.7

degrees Celsius
the units of temperature, written °C. Water freezes at 0 °C. M3.1.1

digit sum
the answer when you add up all the digits of a number. M3.1.4

divisible
can be divided exactly, with nothing left over. M3.1.4

factor
a whole number that divides into another number exactly. M3.1.3

factor pair
two factors that multiply to make the number, such as 4 and 9 for 36. M3.1.3

factor tree
a diagram that splits a number into smaller factors until only primes are left. M3.1.6

highest common factor
the biggest number that is a factor of two numbers (HCF). M3.1.6

index
the small raised number. It says how many of the number to multiply together. M3.1.7

lakh
a name used in some places for 100 000. M3.1.8

lowest common multiple
the smallest number that is *a* multiple of two numbers (LCM). M3.1.6

multiple
a number in the times table of another number. M3.1.3

negative number
a number below zero, written with *a* minus sign. M3.1.1

positive number
a number above zero. M3.1.1

power
a number written with an index, such as 5^2 or 2^3. M3.1.7

power of ten
the number 10 with an index, such as 10^3. It is *a* 1 followed by that many zeros. M3.1.8

prime factors
the factors of *a* number that are prime numbers. M3.1.6

prime number
a number with exactly two factors, 1 and itself. M3.1.5

product
the answer when you multiply numbers. M3.1.2

rise
how many degrees the temperature goes up. M3.1.2

sieve
a way to find primes by crossing out multiples. M3.1.5

square number
a number made by multiplying *a* whole number by itself, such as 25. M3.1.7

square root
the positive number that multiplies by itself to give *a* number. $\sqrt{25} = 5$. M3.1.7

thermometer
a tool that measures temperature. M3.1.1

Answers

M3.1.1. Below zero

1. **−3 °C**

   The liquid is 3 marks below the 0 mark. Each mark is one degree, so the reading is −3 °C.

   - **If you chose A:** The liquid is below the 0 mark, so the number needs a minus sign. A reading without one is above zero. Go to M3.1.1.
   - **If you chose B:** You may have counted the 0 mark as one of the marks. Start counting at the first mark under 0. Go to M3.1.1.

2. **−7, −2, −1, 0, 3**

   On a number line the smallest number is furthest left. The order from left to right is −7, −2, −1, 0, 3.

   - **If you chose A:** You may think −1 is smaller than −2 because 1 is smaller than 2. Below zero it is the other way round. Check on a number line. Go to M3.1.1.
   - **If you chose C:** This list goes from the largest to the smallest. The smallest number goes first, on the left of the number line. Go to M3.1.1.

3. **−8 °C**

   A reading of −8 °C is 8 marks below zero and −3 °C is only 3 marks below. The one further down is colder, and it is the smaller number.

   - **If you chose B:** You may be thinking that a smaller digit means colder. Check on a thermometer. The number −8 is further below zero than −3, so it is colder. Go to M3.1.1.
   - **If you chose C:** A temperature of 8 °C has no minus sign. It is above zero, so it is warmer than both of the others. Go to M3.1.1.

4. **−5**

   A number is smaller when it is further left. The number −5 is left of −2. The numbers −1 and 0 are to the right of it.

   - **If you chose A:** You may think 1 is smaller than 2, so −1 is smaller than −2. On the number line −1 is to the right of −2, so it is bigger. Go to M3.1.1.
   - **If you chose B:** Zero is to the right of every negative number, so it is bigger than −2. Go to M3.1.1.

5. **−1 °C**

   Below freezing means less than 0 °C, so the number must be negative. Only −1 °C is below 0.

   - **If you chose A:** Water freezes at 0 °C. A reading of 0 °C is at the freezing point, not below it. Go to M3.1.1.
   - **If you chose C:** A temperature of 5 °C is above 0 °C. It is warmer than the freezing point. Go to M3.1.1.

6. **−2**

   Left means smaller numbers. From 1 she steps to 0, then −1, then −2.

7. **−6 °C**

   The one further below zero is colder. It has the bigger number after the minus sign.

M3.1.2. Calculating with negative numbers

1. **10 degrees**

   From −6 up to 0 is 6 degrees. From 0 up to 4 is 4 more degrees. 6 + 4 = 10.

   - **If you chose A:** You may have taken 4 away from 6. When one temperature is below zero, count up to zero first and then on to the other. Go to M3.1.2.
   - **If you chose B:** You read the noon temperature. The rise is how far the temperature went up, starting from −6. Go to M3.1.2.

2. **−3**

   Start at −8 and hop 5 to the right. You land on −3.

   - **If you chose A:** You may have added 8 and 5 and kept the minus. Adding 5 moves you to the right, towards zero, not further below it. Go to M3.1.2.
   - **If you chose C:** You started at −8 and hopped 5 to the right. You have not reached 0, so the answer is still below zero and needs a minus sign. Go to M3.1.2.

3. **−6**

   Start at 3 and hop 9 to the left. Three hops reach 0 and six more reach −6.

   - **If you chose B:** You took the smaller number from the bigger one. 3 − 9 hops left past zero, so the answer is below zero. Go to M3.1.2.
   - **If you chose C:** You added 3 and 9. This is a take away, so hop to the left. Go to M3.1.2.

4. **10**

   5 × 2 = 10. The signs are the same, so the answer is positive.

   - **If you chose A:** You may think two minus signs make a negative answer. Same signs give a positive answer. Look at the pattern table again. Go to M3.1.2.
   - **If you chose B:** You may have added −5 and −2. This is a multiplying question, so work out 5 × 2 and then choose the sign. Go to M3.1.2.

5. **−4**

   24 ÷ 6 = 4. The signs are different, so the answer is negative. Check: 6 × −4 = −24.

   - **If you chose A:** You may have added 6 to −24. This is a divide question. Work out 24 ÷ 6 and then choose the sign. Go to M3.1.2.
   - **If you chose C:** The signs are different, one minus and one plus, so the answer is negative. Check: 6 × 4 is 24, not −24. Go to M3.1.2.

6. **9 degrees**

   From −2 up to 0 is 2 degrees. From 0 up to 7 is 7 more degrees. 2 + 7 = 9.

7. **−21**

   7 × 3 = 21. The signs are different, so the answer is negative.

8. **−7**

   The smallest number is the furthest left on a number line. That number is −7. It is left of −2, 0 and 4.

   - **If you chose A:** Zero is bigger than every negative number. The smallest number is the one furthest to the left. Go to M3.1.1.
   - **If you chose C:** You may think −2 is the smallest because 2 is the smallest digit. A number further left is smaller, and −7 is further left than −2. Go to M3.1.1.

M3.1.3. Multiples and factors

1. **1, 2, 4, 5, 10 and 20**

   The pairs are 1 × 20, 2 × 10 and 4 × 5. Write the numbers in order.

   - **If you chose A:** You left out 1 and 20. Every number has 1 and itself as factors, because 1 × 20 = 20. Go to M3.1.3.
   - **If you chose B:** These are multiples of 20, not factors. A factor divides into 20, so it is never bigger than 20. Go to M3.1.3.

2. **6**

   42 ÷ 6 = 7 with nothing left over, so 6 is a factor. The other three leave a remainder.

   - **If you chose A:** 42 is between 36 and 45 in the 9 times table, so it is not in it. 42 ÷ 9 = 4 r 6, so 9 is not a factor. Go to M3.1.3.
   - **If you chose B:** 15 is smaller than 42, but 42 ÷ 15 leaves a remainder. A factor must divide exactly. Go to M3.1.3.
   - **If you chose D:** 42 is between 40 and 48 in the 8 times table, so it is not in it. 42 ÷ 8 = 5 r 2, so 8 is not a factor. Go to M3.1.3.

3. **1 × 28, 2 × 14 and 4 × 7**

   28 ÷ 1 = 28, 28 ÷ 2 = 14 and 28 ÷ 4 = 7. 3, 5 and 6 leave a remainder. After 4 the pairs repeat.

   - **If you chose B:** You left out the pair 1 × 28. Every number makes a pair with 1. Go to M3.1.3.
   - **If you chose C:** 3 × 9 = 27, not 28. Check that each pair multiplies to exactly 28. Go to M3.1.3.

4. **7, 14, 21, 28 and 35**

   Count in 7s: 7, 14, 21, 28, 35. These are the first five numbers in the 7 times table.

   - **If you chose A:** You started at 14. The first multiple of 7 is 7 itself, because 1 × 7 = 7. Go to M3.1.3.
   - **If you chose B:** You added 10 each time. Multiples of 7 go up by 7 each time. Go to M3.1.3.

5. **30**

   Only 30 is in both tables: 3 × 10 = 30 and 5 × 6 = 30.

   - **If you chose A:** 25 is in the 5 times table, but 3 does not divide it: 25 ÷ 3 leaves 1. A common multiple must be in both tables. Go to M3.1.3.
   - **If you chose C:** 20 is in the 5 times table, but 3 does not divide it: 20 ÷ 3 leaves 2. A common multiple must be in both tables. Go to M3.1.3.

6. **6, 12, 18, 24 and 30**

   Count in 6s: 6, 12, 18, 24, 30.

7. **8 factors**

   The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24. That is 8 numbers.

8. **42**

   Two numbers with the same sign give a positive answer. 6 × 7 = 42, so −6 × −7 = 42.

M3.1.4. Quick tests for dividing

1. **1 115**

   1 115 ends in 5, so it is divisible by 5. 1 062 ends in 2 and 2 004 ends in 4.

   - **If you chose B:** You may have looked at the digit sum, 6. The digit sum is for 3 and 9. For 5, look at the last digit, which is 4. Go to M3.1.4.
   - **If you chose C:** This number ends in 2. The last digit must be 0 or 5 for a number to divide by 5. Look at the tests again. Go to M3.1.4.

2. **3 840**

   3 840 ends in 0, so it is a multiple of 10. 3 840 = 384 × 10.

   - **If you chose A:** This number ends in 4, so it is divisible by 2. But a number divisible by 10 must end in 0. Go to M3.1.4.
   - **If you chose C:** This number ends in 5, so it is divisible by 5. But a number divisible by 10 must end in 0. Go to M3.1.4.

3. **5 817**

   5 + 8 + 1 + 7 = 21, and 21 = 3 × 7. So 5 817 is divisible by 3. Check: 5 817 ÷ 3 = 1 939.

   - **If you chose B:** This number ends in 3, but the last digit does not decide it. The digit sum is 17, which is not in the 3 times table. Go to M3.1.4.
   - **If you chose C:** This number ends in 0, so it is divisible by 10. For 3 you need the digit sum, and 14 is not in the 3 times table. Go to M3.1.4.

4. **6 723**

   6 + 7 + 2 + 3 = 18, and 18 = 2 × 9. So 6 723 is divisible by 9. Check: 6 723 ÷ 9 = 747.

   - **If you chose A:** The digit sum is 21. That is in the 3 times table, so 6 726 is divisible by 3. But 21 is not in the 9 times table, so it is not divisible by 9. Go to M3.1.4.
   - **If you chose B:** The digit sum is 16. That is not in the 9 times table, so 6 703 is not divisible by 9. Go to M3.1.4.

5. **4 536**

   4 536 ends in 6, so 2 divides it. Its digit sum is 4 + 5 + 3 + 6 = 18, so 3 divides it too.

   - **If you chose A:** The digit sum is 15, so it is divisible by 3. But it ends in 3, so it is not divisible by 2. Go to M3.1.4.
   - **If you chose C:** This number ends in 8, so it is divisible by 2. But its digit sum is 20, which is not in the 3 times table. Go to M3.1.4.

6. **21**

   5 + 8 + 1 + 7 = 21.

7. **2**

   5 + 2 = 7. The next number in the 9 times table is 9, and 7 + 2 = 9. So the digit is 2, and 522 = 9 × 58.

8. **1, 2, 3, 6, 9 and 18**

   The factor pairs of 18 are 1 × 18, 2 × 9 and 3 × 6. Write the numbers in order.

M3.1.5. Prime numbers

1. **No, 51 = 3 × 17.**

   The digit sum is 5 + 1 = 6, which is in the 3 times table. So 3 divides 51, and 51 = 3 × 17.

   - **If you chose A:** Many odd numbers are not prime. The digit sum of 51 is 6. That is in the 3 times table, so 3 divides 51. Go to M3.1.5.
   - **If you chose B:** You left out 3. Test 2, 3, 5 and 7. The digit sum of 51 is 6, so 3 divides 51. Go to M3.1.5.

2. **59**

   Test 2, 3, 5 and 7 on 59. None divides it exactly. The other three each have a factor from that list.

   - **If you chose B:** 65 ends in 5, so 5 divides it. 65 = 5 × 13. Go to M3.1.5.
   - **If you chose C:** You may think every odd number is prime. The digit sum of 57 is 12, so 3 divides it. 57 = 3 × 19. Go to M3.1.5.
   - **If you chose D:** 63 is in the 7 times table and in the 9 times table. It has more than two factors. Go to M3.1.5.

3. **The number 2 is the only even prime.**

   Every even number after 2 divides by 2, so it has at least three factors. Only 2 has just two factors, 1 and 2.

   - **If you chose B:** The number 2 is even, and its only factors are 1 and 2. So 2 is an even prime. Go to M3.1.5.
   - **If you chose C:** The number 1 has only one factor, itself. A prime has exactly two factors, so 1 is not prime. Go to M3.1.5.

4. **The multiples of 5**

   The first number not crossed out after 3 is 5. It is prime. Ring 5, then cross out the other multiples of 5.

   - **If you chose A:** Every multiple of 4 is also a multiple of 2, so it is already crossed out. Go to M3.1.5.
   - **If you chose B:** Every multiple of 6 is also a multiple of 2, so it is already crossed out. Go to M3.1.5.

5. **The next prime is 11, and 11 × 11 = 121 is more than 100.**

   A composite number up to 100 has a factor of 10 or less, other than 1. Every such factor is 2, 3, 5 or 7, or divides by one of them.

   - **If you chose A:** That sounds right, but it is not the reason. The sieve stops at 7 because 11 × 11 = 121 is more than 100. Go to M3.1.5.
   - **If you chose C:** There are many primes after 7, such as 11, 13 and 17. They just have no new multiples to cross out below 100. Go to M3.1.5.

6. **7**

   2, 3 and 5 do not divide 91, but 7 × 13 = 91.

7. **10 primes**

   The primes are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29. That makes 10.

8. **25 primes**

   Count the rings row by row: 4, 4, 2, 2, 3, 2, 2, 3, 2 and 1. They add up to 25.

9. **Yes, because the digit sum is 15.**

   8 + 1 + 2 + 4 = 15, and 15 = 3 × 5. So 8 124 is divisible by 3.

M3.1.6. Prime factors, common factors and multiples

1. **2 × 2 × 3 × 3**

   2 × 2 × 3 × 3 = 36, and 2 and 3 are both prime. The other two lists have numbers that can still be split.

   - **If you chose A:** These multiply to 36, but 4 and 9 are not prime. Split 4 into 2 × 2 and 9 into 3 × 3. Go to M3.1.6.
   - **If you chose B:** 18 is not prime. Split it: 18 = 2 × 9, and 9 = 3 × 3. Keep going until every number is prime. Go to M3.1.6.

2. **2 × 3 × 3 × 5**

   90 = 9 × 10. Then 9 = 3 × 3 and 10 = 2 × 5. In order, the primes are 2, 3, 3 and 5.

   - **If you chose A:** 9 is not prime. Split it into 3 × 3. Every number in the list must be prime. Go to M3.1.6.
   - **If you chose C:** 9 and 10 multiply to 90, but neither is prime. Split 9 into 3 × 3 and 10 into 2 × 5. Go to M3.1.6.

3. **9**

   The branches from 72 must multiply back to 72. 8 × ? = 72, so ? = 72 ÷ 8 = 9. It splits into 3 × 3, which is 9.

   - **If you chose B:** You took 8 away from 72. The two branches multiply to give 72, so 8 × ? = 72. Go to M3.1.6.
   - **If you chose C:** You added 8 to 72. The branches of a factor tree multiply back to the number above, they do not add. Go to M3.1.6.

4. **6**

   18 = 2 × 3 × 3 and 30 = 2 × 3 × 5. They share 2 and 3. 2 × 3 = 6.

   - **If you chose A:** 3 is a common factor of 18 and 30, but it is not the highest. 6 is bigger and divides both. Go to M3.1.6.
   - **If you chose B:** 90 is the lowest common multiple of 18 and 30. A factor of 18 can never be bigger than 18. Go to M3.1.6.

5. **20**

   Multiples of 10 are 10, 20, 30. Multiples of 4 are 4, 8, 12, 16, 20. The first one in both lists is 20.

   - **If you chose A:** 40 is a common multiple of 4 and 10, but it is not the lowest. 20 is in both tables and is smaller. Go to M3.1.6.
   - **If you chose C:** 2 is the highest common factor of 4 and 10. A multiple is never smaller than the number itself. Go to M3.1.6.

6. **8**

   16 = 2 × 2 × 2 × 2 and 40 = 2 × 2 × 2 × 5. They share 2, 2 and 2. 2 × 2 × 2 = 8.

7. **36**

   Multiples of 12: 12, 24, 36. Multiples of 18: 18, 36. The first one in both lists is 36.

8. **No, 87 = 3 × 29.**

   The digit sum is 8 + 7 = 15, which is in the 3 times table. So 3 divides 87, and 87 = 3 × 29.

M3.1.7. Squares, cubes and roots

1. **$2^5 = 32$**

   There are five 2s, so the index is 5. 2 × 2 = 4, 4 × 2 = 8, 8 × 2 = 16 and 16 × 2 = 32.

   - **If you chose A:** You swapped the 2 and the 5. The number being multiplied is 2, and it is used five times, so the index is 5. Go to M3.1.7.
   - **If you chose B:** The small number does not multiply the 2. It counts how many 2s are multiplied together. Go to M3.1.7.

2. **49**

   7^2 means 7 × 7, and 7 × 7 = 49.

   - **If you chose A:** You added 7 and 2. A power is a multiplying, not an adding. 7^2 = 7 × 7. Go to M3.1.7.
   - **If you chose C:** You worked out 7 × 2. The index 2 means two 7s multiplied together: 7 × 7. Go to M3.1.7.

3. **64**

   4 × 4 = 16, then 16 × 4 = 64. So 4^3 = 64.

   - **If you chose B:** You may have worked out 4 × 3. The index 3 means three 4s multiplied together: 4 × 4 × 4. Go to M3.1.7.
   - **If you chose C:** You stopped after two 4s. That is 4^2. For 4^3 you multiply by 4 once more. Go to M3.1.7.

4. **9**

   9 × 9 = 81, so the square root of 81 is 9. It is the number of tiles along one side.

   - **If you chose A:** You divided 81 by 3. The square root is the number of tiles along one side, which multiplies by itself to give 81. Go to M3.1.7.
   - **If you chose B:** 3 × 3 × 3 × 3 = 81, so 3 is the number in 3^4 = 81. The square root uses the number twice: 9 × 9 = 81. Go to M3.1.7.

5. **4**

   4 × 4 × 4 = 64, so the cube root of 64 is 4. It is the number of small cubes along one edge.

   - **If you chose A:** You halved 64. The cube root is the number of small cubes along one edge, used three times in a multiplying. Go to M3.1.7.
   - **If you chose C:** 8 × 8 = 64, so 8 is the square root of 64. The cube root uses the number three times: 4 × 4 × 4 = 64. Go to M3.1.7.

6. **81**

   9^2 means 9 × 9, and 9 × 9 = 81.

7. **10**

   10 × 10 = 100, so the square root of 100 is 10.

8. **$72 = 2 \times 2 \times 2 \times 3 \times 3$**

   $72 = 8 \times 9$, and $8 = 2 \times 2 \times 2$ and $9 = 3 \times 3$. With powers this is $2^3 \times 3^2$.

M3.1.8. Powers of ten

1. **1 000**

   $10^3 = 10 \times 10 \times 10 = 1\,000$. It has 3 zeros.

   - **If you chose A:** You may have multiplied 10 by 3. The index 3 says there are three 10s multiplied together, and three zeros in the answer. Go to M3.1.8.
   - **If you chose B:** You may have written the 3 and then two zeros. The index is not a digit of the answer. It counts the zeros after the 1. Go to M3.1.8.

2. **10^5**

   There are five zeros after the 1, so the index is 5.

   - **If you chose A:** You counted only four zeros. Look again: 100 000 has five zeros after the 1. Go to M3.1.8.
   - **If you chose C:** You counted the digits. 100 000 has six digits, but the index counts the zeros after the 1, and there are five. Go to M3.1.8.

3. **10 000 000**

   One crore is ten million, 10 000 000. It is 1 followed by seven zeros, or 10^7.

   - **If you chose B:** That is one million, which is ten lakh. One crore has seven zeros. Go to M3.1.8.
   - **If you chose C:** That is one lakh, which is one hundred thousand. One crore is 100 times as big. Go to M3.1.8.

4. **4×10^6**

   $4 \times 10^6 = 4\,000\,000$ and $9 \times 10^5 = 900\,000$. 4 000 000 has seven digits and 900 000 has six.

   - **If you chose A:** You looked at the 9, which is bigger than 4. But 10^6 is ten times as big as 10^5. Write both in full: 4 000 000 and 900 000. Go to M3.1.8.
   - **If you chose B:** The index is not multiplied by the front number. It counts the zeros. Write both in full: 4 000 000 and 900 000. Go to M3.1.8.

5. **100 lakh**

   10 000 000 ÷ 100 000 = 100. In powers, $10^7 \div 10^5 = 10^2$, which is 100.

   - **If you chose A:** 1 000 lakh is 100 000 000, which is ten times too big. Divide 10 000 000 by 100 000. Go to M3.1.8.
   - **If you chose C:** Ten lakh is 1 000 000, which is one million. One crore is ten million, 10 times as big. Go to M3.1.8.

6. **10 000**

   10^4 is a 1 followed by 4 zeros. That is 10 × 10 × 10 × 10 = 10 000.

7. **30 000**

   10^4 = 10 000, so $3 \times 10^4 = 3 \times 10\,000 = 30\,000$.

8. **11**

   11 × 11 = 121, so the square root of 121 is 11.

About this book

For learners who know the times tables up to 10 and can add, take away, multiply and divide whole numbers. In many schools this fits children of about 11 to 13, and older learners who skipped negative numbers or primes can start here too. Each chapter takes one or two short sessions. A learner can work alone: every exercise has its answer and the reason for it.

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