●●●○○ Level 3 · Mathematics M3.8

# Letters for numbers and simple equations

## Use letters to stand for numbers, tidy up expressions, and solve simple equations.

Start with a box of beans and write what you do not know as a letter. Put numbers in, collect like terms, use brackets, and keep a balance level to solve equations. Then write your own equations and find the rule of a sequence.

**YOU ARE READY FOR THIS BOOK IF**

You can work out 3 + 4 × 2 in the right order.

| | |
|---|---|
| **One step up** | M4.8 Linear and simultaneous equations |
| **This book** | **M3.8 Letters for numbers and simple equations** |
| **One step down** | M2.8 Patterns and rules |

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Key words, A to Z


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M3.8.1 CHAPTER 1

# Letters for unknown numbers

> **Ravi has a closed box of beans and 5 loose beans. How can he write down all his beans?**

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**YOUR GUESS**

Pick one before you read on. It is fine to be wrong.

- ☐ **A** He cannot write it until he opens the box and counts.
- ☐ **B** He writes n + 5, where n is the number of beans in the box.
- ☐ **C** He writes 5, because the box does not matter.

**IF THIS FEELS HARD, READ THIS FIRST**

- Level 2, Patterns and rules, M2.8.3. A rule such as times 3, then add 1 is the same idea as a letter.
- Level 3, Negative numbers, factors and powers, M3.1.7. If 5 squared is new to you, read this first.

**BY THE END YOU CAN**

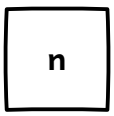
- Write 'a number add 5' as n + 5
- Write 3 × n as 3n and n × n as $n^2$

Ravi has a closed box of beans. He also has 5 loose beans. He cannot see how many beans are in the box.

In algebra, a letter stands for a number we do not know. We call it an **unknown**. Ravi writes n for the number of beans in the box.

DRAW IT FIRST

## The box and 5 more beans




box: n beans

**1** Draw the closed box. Write n on it. It holds n beans.




box: n beans 5 loose beans

**2** Draw the 5 loose beans beside it, with a plus sign.



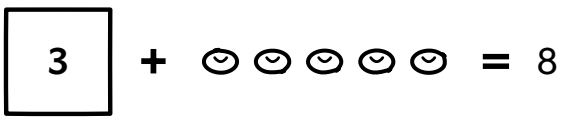

box: n beans 5 loose beans

**n + 5**

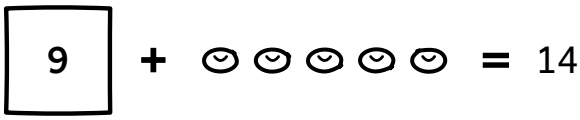
**3** Write what you see: n + 5.

Numbers, letters and signs written together are an **expression**. Ravi has the expression n + 5.

The letter is not a puzzle. The box holds a real number. It may be 3 one day and 9 the next. The letter n stands for whichever it is.




n is 3, so n + 5 is 8




n is 9, so n + 5 is 14

The same expression, n + 5, fits both boxes.

WORKED EXAMPLE

**Ana has n pencils in a tin. Her brother has 3 more than Ana. How many pencils does her brother have?**

1. Ana has n pencils.
2. 3 more means we add 3.
3. Her brother has n + 3 pencils.
4. Try n = 10. Ana has 10, and her brother has 10 + 3 = 13.

**Answer:** n + 3

To take beans away we use a minus sign. Amina has n stickers and gives 3 away. She has n − 3 left.




n − 3

The bar is n. Its length does not tell you n. The crossed squares are the 3 taken away.

Now Ravi has 3 boxes. Each box holds the same number of beans. He could write n + n + n.

There is a shorter way. We write 3n. There is no sign between the 3 and the n. The 3 in front is the **coefficient**. It counts the boxes.

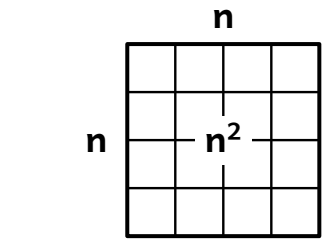
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SEE IT THREE WAYS

| With things | As a picture | In symbols |
|---|---|---|
| Put 3 cups on a table. Put the same number of beans in each cup, and do not count them. |   | **n + n + n = 3 × n = 3n.** |

---

What is n times n? It is n multiplied by itself. We write $n^2$ and say n **squared**.




$n^2$ is 16 squares




2n is 8 squares

Here n is 4. The square $n^2$ has 16 small squares. The two bars 2n have only 8.

**WORKED EXAMPLE**

**A box holds n beans. Write the beans in 3 boxes. Then write n × n. Try n = 4.**

1. 3 boxes of n beans is 3 × n. We write 3n.
2. n × n means n multiplied by itself. We write $n^2$.
3. Try n = 4. Then 3n = 3 × 4 = 12.
4. And $n^2 = 4 \times 4 = 16$.

**Answer:** 3n and $n^2$. When n = 4, they are 12 and 16.

TRY IT

## A cup you cannot see into

**You need:** a cup or a small tin, dry beans, stones or seeds, a friend, paper and a pencil.

1. Ask your friend to put some beans in the cup and cover it. Do not look.
2. Put 5 more beans on the table, next to the cup.
3. Write n + 5 on the paper. Here n means the beans in the cup.
4. Now count the beans in the cup, then count them all. Is the total n plus 5?

**What you will see:** Whatever the cup holds, the total is that number plus 5. The letter n is a plan that works for any number.

---

If you cannot try it, this is what happens.




n + 5

A WRONG IDEA MANY PEOPLE HAVE

### ~~$n^2$ means the same as 2n.~~

**Why it feels right:** Both have a 2 in them, and both seem to say "twice".

**The test that breaks it:** Let n be 4. Then 2n is 4 + 4 = 8. But $n^2$ is 4 × 4 = 16. The square picture has 16 small squares.

**What is true:** $n^2$ is n × n. The small 2 says n is used two times in a product. 2n is n + n.

KEY WORDS

**unknown**
a number we do not know yet, written as a letter.

**expression**
numbers, letters and signs written together, such as n + 5.

**coefficient**

the number in front of *a* letter. In 3n it is 3.

**squared**
multiplied by itself. n squared is n × n, written $n^2$.

BACK TO YOUR GUESS

**The answer is B: He writes n + 5, where n is the number of beans in the box.**

We can write *a* number we do not know *as a* letter. Ravi has n + 5 beans. This is true whatever the box holds.

---

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Ravi has one box of beans and 4 loose beans. The bar is the box and the squares are the loose beans. The length of the bar does not tell you n. Which expression shows all his beans?




   **A** n + 4

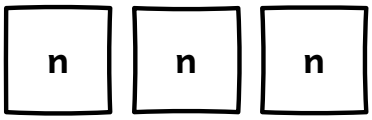
   **B** 4n

   **C** n4

   **D** n − 4

   Answer at the back of the book.

2. Tariq has 3 boxes. Every box holds the same number of beans, n. How many beans does he have?




**A** n + 3

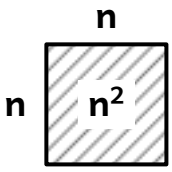
**B** n3

**C** 3n

**D** 3 + n

Answer at the back of the book.

3. What does $n^2$ mean? The picture shows a square with side n.




**A** n × 2

**B** n + n

**C** n + 2

**D** n × n

Answer at the back of the book.

4. Mei says that $n^2$ and 2n are the same. Test her with n = 5. Which sentence is true?

**A** 2n is 10 and $n^2$ is 10

**B** 2n is 10 and $n^2$ is 25

**C** 2n is 25 and $n^2$ is 10

**D** 2n is 25 and $n^2$ is 25

Answer at the back of the book.

5. Amina has n stickers. She gives 3 away. The bar is her stickers and the crossed squares are the 3 she gives away. How many stickers are left?




**A** n − 3

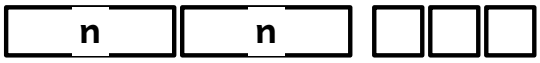
**B** n + 3

**C** 3 − n

**D** 3n

Answer at the back of the book.

6. Look at the picture. Each bar is a box with n beans. Each square is one loose bean. Which expression shows all the beans?




**A** 5n

**B** n + 5

**C** 2n + 3

**D** 2 + 3n

Answer at the back of the book.

7. A box holds n beans. There are also 5 loose beans. If n is 9, how many beans are there in all?

Answer at the back of the book.

8. What is $n^2$ when n is 6?

Answer at the back of the book.

**KEEP IT**

Copy this into your exercise book.

**A letter stands for an unknown number. n + 5 is a box and 5 more. 3n is 3 × n. $n^2$ is n × n, and it is not 2n.**

**Draw:** Draw a box marked n beside 5 beans, and write n + 5. Draw a square with side n and write $n^2$ in it.

# Putting numbers in

> **Leila has 3 boxes of beans and 2 loose beans. Each box holds 4 beans. How many beans does she have?**

---

**YOUR GUESS**

Pick one before you read on. It is fine to be wrong.

- ☐ **A** She has 36. The boxes are 34, because 3n with n = 4 puts 3 and 4 side by side. Then add 2.
- ☐ **B** She has 14. The boxes are 3 × 4 = 12, because 3n means 3 times n. Then add 2.
- ☐ **C** She has 9. She adds every number she sees: 3 + 4 + 2.

**IF THIS FEELS HARD, READ THIS FIRST**

- Level 3, Long multiplication and division, M3.2.6. Multiplying comes before adding. If this is new, read this first.
- Level 2, Patterns and rules, M2.8.3. A number machine puts a number in and gives a number out.

**BY THE END YOU CAN**

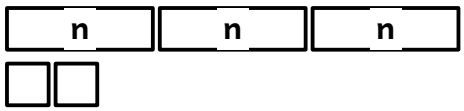
- Find the value of 3n + 2 when n = 4
- Use the formula P = 2l + 2w for a rectangle

In the last chapter, n stood for the beans in a box. Leila has 3 boxes and 2 loose beans. She has 3n + 2 beans.

Now we are told that each box holds 4 beans. Then we can find the number of beans. We put 4 in place of n. This is called to **substitute**. The number we get is the **value** of the expression.

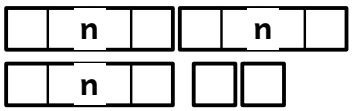
DRAW IT FIRST

## Cut each bar into 4 squares



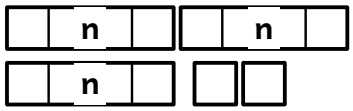

3n + 2

**1** Draw 3 bars of n and 2 squares. This is 3n + 2. The length of a bar does not tell you n.




n = 4

**2** Now we know n is 4, so each bar is drawn 4 squares long. Cut every bar into 4 squares.




n = 4

**3 × 4 + 2 = 14 squares**

**3** Count the squares: 3 × 4 + 2 = 14.

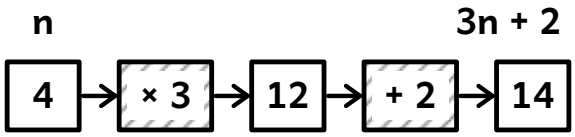
Take care with 3n. It means 3 × n. It does not put 3 and 4 side by side. With n = 4, 3n is 3 × 4 = 12, not 34.

WORKED EXAMPLE

**Find the value of 3n + 2 when n = 4.**

1. Write 3n as 3 × n.
2. Put in n = 4. This gives 3 × 4 + 2.
3. Multiply first: 3 × 4 = 12.
4. Then add: 12 + 2 = 14.

**Answer:** 14




Put in 4. The machine multiplies by 3, then adds 2.

WORKED EXAMPLE

**Find the value of $n^2 + 1$ when n = 5.**

1. Write $n^2$ as n × n.
2. Put in n = 5. This gives 5 × 5 + 1.
3. Multiply first: 5 × 5 = 25.
4. Then add: 25 + 1 = 26.

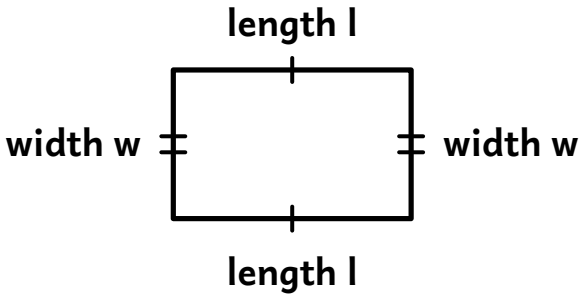
**Answer:** 26

A table helps when you want many values. Put in n = 1, then 2, then 3, and so on.

| n | 3n + 2 |
|---|---|
| 1 | 5 |
| 2 | 8 |
| 3 | 11 |
| 4 | 14 |
| 5 | 17 |

Each time n goes up by 1, the value goes up by 3.

A **formula** is a rule written with letters. The distance all the way round a rectangle is its perimeter. A rectangle has 2 lengths and 2 widths. So P = 2l + 2w.



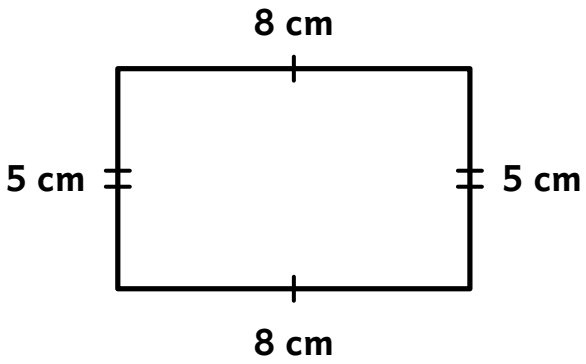

P is the perimeter, l is the length and w is the width.

WORKED EXAMPLE

**A rectangle is 8 cm long and 5 cm wide. Find its perimeter.**

1. The formula is P = 2l + 2w.
2. Two lengths make 2 × 8 = 16 cm.
3. Two widths make 2 × 5 = 10 cm.
4. Add them: 16 + 10 = 26 cm.

**Answer:** 26 cm




Go round: 8 + 5 + 8 + 5 = 26 cm. The formula gives the same.

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**SEE IT THREE WAYS**

| With things | As a picture | In symbols |
| --- | --- | --- |
| Put 4 beans in each of 3 cups. Add 2 loose beans. Count every bean. |   | **3n + 2 with n = 4 is 3 × 4 + 2 = 14.** |

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TRY IT

## Measure the way round a book

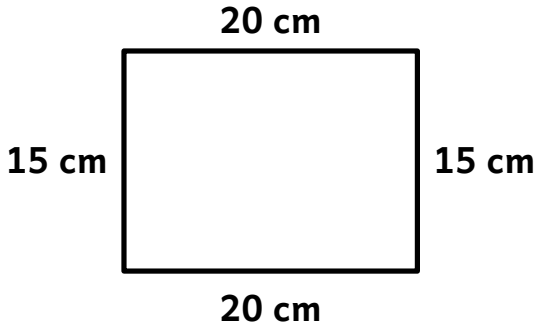
**You need:** a book or a sheet of paper, a ruler or a piece of string, paper and a pencil.

1. Measure the length of the book in cm. Call it l.
2. Measure the width in cm. Call it w.
3. Work out 2l + 2w.
4. Now lay string round the edge. Measure the string. Is it the same?

**What you will see:** A book 20 cm long and 15 cm wide gives 2 × 20 + 2 × 15 = 70 cm. Your numbers will differ. The string agrees with the formula, to within a few millimetres.

---

If you cannot try it, this is what happens.




A WRONG IDEA MANY PEOPLE HAVE

**~~When n = 4, the expression 3n means 34.~~**

**Why it feels right:** In a number like 34, two digits stand side by side. A letter beside a number looks the same.

**The test that breaks it:** Take 3 boxes with 4 beans in each. Count them: 4 + 4 + 4 = 12 beans, not 34.

**What is true:** A number next to a letter means multiply. 3n is 3 × n, so 3n is 12 when n is 4.

KEY WORDS

**substitute**
to put a number in place of a letter.

**value**
the number an expression gives when you substitute.

**formula**
a rule written with letters, such as P = 2l + 2w.

BACK TO YOUR GUESS

**The answer is B: She has 14. The boxes are 3 × 4 = 12, because 3n means 3 times n. Then add 2.**

3n means 3 times n, so the boxes hold 3 × 4 = 12 beans. With 2 loose beans that is 14. It would be 34 only if the 3 and the 4 were a two digit number.

---

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Find the value of 3n + 2 when n = 5.

   **A** 37

   **B** 21

   **C** 10

   **D** 17

   Answer at the back of the book.

2. Find the value of 20 − 2n when n = 4.

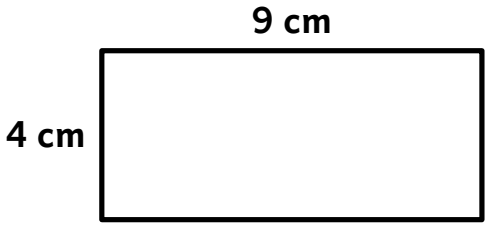
   **A** 12

   **B** 16

   **C** 72

   Answer at the back of the book.

3. Use P = 2l + 2w. The rectangle is 9 cm long and 4 cm wide. What is its perimeter?




**A** 36 cm

**B** 13 cm

**C** 26 cm

**D** 22 cm

Answer at the back of the book.

4. Kofi uses P = 2l + 2w with l = 7 and w = 3. Which working is right?

**A** 27 + 23 = 50

**B** 7 + 3 = 10

**C** 2 × 7 × 3 = 42

**D** 2 × 7 + 2 × 3 = 20

Answer at the back of the book.

5. Let n = 3. Which is bigger, 2n or n + 4?

**A** 2n is bigger, because multiplying always gives more.

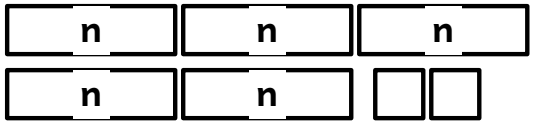
**B** n + 4 is bigger. It is 7, and 2n is 6.

**C** They are equal.

Answer at the back of the book.

6. **From an earlier chapter**

The picture shows 5 boxes of n beans and 2 loose beans. Which expression is it?




**A** n + 7

**B** 5 + 2n

**C** 5n + 2

**D** 52n

Answer at the back of the book.

7. Find the value of 4n + 1 when n = 6.

Answer at the back of the book.

8. A rectangle is 12 cm long and 7 cm wide. Use P = 2l + 2w to find its perimeter.

Answer at the back of the book.

**KEEP IT**

Copy this into your exercise book.

**Put the number in place of the letter. 3n means 3 × n, so n = 4 gives 12, not 34. Multiply before you add. P = 2l + 2w.**

**Draw:** Draw a machine: 4, then × 3, then 12, then + 2, then 14. Draw a rectangle and write l, w, l, w on its sides.

# Collecting like terms

> **Sipho has 3 boxes of mangoes and 2 boxes of limes. A mango box holds *a* mangoes. A lime box holds b limes. He gets 5 more mango boxes and gives away 1 lime box. How many pieces of fruit does he have?**

---

**YOUR GUESS**

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 9*a*b. He adds 3 + 2 + 5 − 1 = 9 and keeps both letters.
- ☐ **B** 8*a* + b. The mango boxes are counted together, and the lime boxes are counted together.
- ☐ **C** 8*a* + 2b. Giving away *a* box does not change the limes.

**IF THIS FEELS HARD, READ THIS FIRST**

- Level 3, Negative numbers, factors and powers, M3.1.2. Taking away *a* b is the same as adding minus b. If that is new, read this first.

**BY THE END YOU CAN**

- Say which terms are like terms and which are not
- Simplify 3*a* + 2b + 5*a* − b

Sipho has 3 boxes of mangoes and 2 boxes of limes. Each mango box holds *a* mangoes. Each lime box holds b limes. He has 3*a* + 2b pieces of fruit.

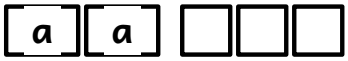
Each part of an expression, with the sign in front of it, is a **term**. The expression 3*a* + 2b has two terms. They are 3*a* and 2b.




2*a* and 3*a* are like terms




2*a* and 3b: not like terms




2*a* and 3: not like terms

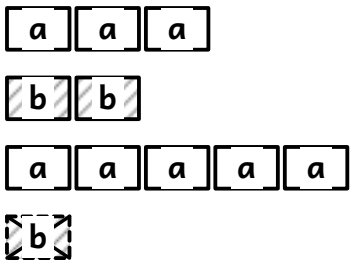
The shading tells the letters apart. The length of *a* bar is only for the picture.

The numbers *a* and b may be different, so 3*a* and 2b cannot be joined. But 3*a* and 5*a* are both *a* number of *a*. Terms with the same letter part, such as 3*a* and 5*a*, are **like terms**. The letter part of $a^2$ is not the same as *a*.

We can count like terms together. We cannot count terms with different letters together. A plain number is not like *a* term with *a* letter.

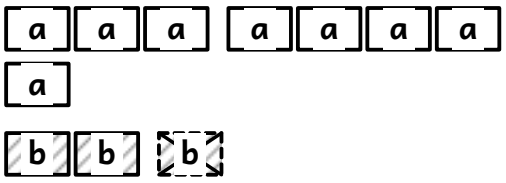
DRAW IT FIRST

## Group the bars that are alike



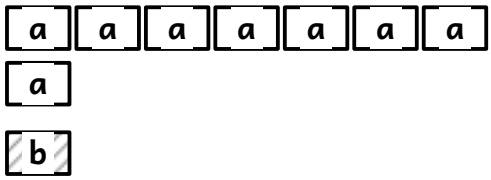

**3a + 2b + 5a − b**

**1** Draw one row for each term: 3a + 2b + 5a − b. The crossed bar is taken away.




**like terms together**

**2** Move the bars that are alike next to each other.




**8a + b**

**3** Count each kind. There are 8 bars of a. The crossed bar cancels 1 of the 2 bars of b, so 1 bar of b is left.

We add the numbers in front of the letters and keep the letter. 3a + 5a = 8a. And 2b − b = b. We write b, not 1b.

**WORKED EXAMPLE**

**Simplify $3a + 2b + 5a - b$.**

1. The $a$ terms are $3a$ and $5a$. The b terms are $2b$ and $-b$.
2. Count the $a$ bars: $3 + 5 = 8$.
3. Count the b bars: $2 - 1 = 1$.
4. So $3a + 2b + 5a - b = 8a + b$.

**Answer:** $8a + b$

To **simplify** an expression is to write it in its shortest form. We collect the like terms to do this.

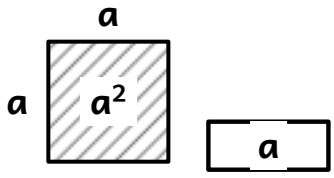
**WORKED EXAMPLE**

**Simplify $4x + 3 + 2x - 1$.**

1. The x terms are $4x$ and $2x$. The plain numbers are 3 and $-1$.
2. Count the x bars: $4 + 2 = 6$.
3. Add the plain numbers: $3 - 1 = 2$.
4. So $4x + 3 + 2x - 1 = 6x + 2$.

**Answer:** $6x + 2$

The term $a^2$ is not like the term $a$. The term $a^2$ is a square. The term $a$ is a bar. The two shapes are different.




$a^2$ and $a$ are not like terms.

WORKED EXAMPLE

**Check that 8*a* + b is the same as 3*a* + 2b + 5*a* − b when *a* = 2 and b = 3.**

1. Put *a* = 2 and b = 3 into 3*a* + 2b + 5*a* − b.
2. The *a* terms: 3 × 2 + 5 × 2 = 16.
3. The b terms: 2 × 3 − 3 = 3.
4. Add them: 16 + 3 = 19.
5. Now 8*a* + b gives 8 × 2 + 3 = 19. The two agree.

**Answer:** 19 and 19. They agree.

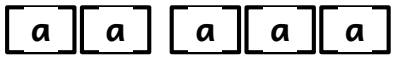
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SEE IT THREE WAYS

**With things**

Put 2 cups of beans and 3 more cups of beans in *a* row. Every cup holds the same number of beans, *a*. Count the cups. 5 cups of *a* beans hold 5*a* beans.

**As *a* picture**




2*a* + 3*a* = 5*a*

**In symbols**

**2*a* + 3*a* = 5*a*. Add the numbers in front. The letter stays.**

---

TRY IT

## Cups and bowls

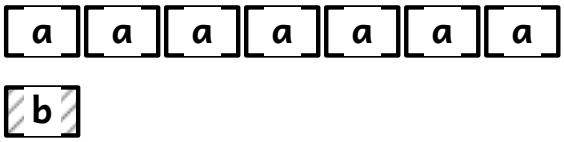
**You need:** 3 cups and 2 bowls, beans or stones to put in them, paper and a pencil.

1. Put the same number of beans in each cup. Put the same number of stones in each bowl. Do not count them.
2. Let a be the beans in a cup and b the stones in a bowl. Put 3 cups and 2 bowls on the table.
3. Add 4 more cups. Take away 1 bowl.
4. Count the cups and the bowls. Write the number of beans and stones with a and b.

**What you will see:** There are 7 cups of a beans and 1 bowl of b stones. This is 3a + 2b + 4a − b = 7a + b in all. The numbers a and b may differ, so they stay apart.

---

If you cannot try it, this is what happens.




7a + b

A WRONG IDEA MANY PEOPLE HAVE

**~~Always add the numbers in front. So 3a + 2b = 5ab.~~**

**Why it feels right:** 3 + 2 is 5, and it seems tidy to keep both letters.

**The test that breaks it:** Let a be 10 and b be 1. Then 3a + 2b is 30 + 2 = 32. But 5ab is 5 × 10 × 1 = 50. They are not equal.

**What is true:** Only like terms are joined. 3a + 2b has two different kinds of term, so it stays 3a + 2b.

KEY WORDS

**term**
one part of an expression, with the sign in front of it.

**like terms**
terms with the same letter part, such as $3a$ and $5a$.

**simplify**
to write an expression in its shortest form.

BACK TO YOUR GUESS

**The answer is B: $8a + b$. The mango boxes are counted together, and the lime boxes are counted together.**

The numbers $a$ and b may be different, so $3a$ and $2b$ cannot be joined. He has $3 + 5 = 8$ boxes of $a$ mangoes and $2 - 1 = 1$ box of b limes. That is $8a + b$ pieces of fruit.

---

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which pair is a pair of like terms?

   **A** $4x$ and $7y$

   **B** $4x$ and $7x$

   **C** $4x$ and $7$

   **D** $4x$ and $7x^2$

   Answer at the back of the book.

2. Simplify $5a + 3a$.

   **A** $8a$

   **B** $15a$

   **C** $8$

   **D** $8a^2$

   Answer at the back of the book.

3. Simplify $4x + 3y + 2x - 2y$.

   **A** $6x + 5y$

   **B** $6x + y$

   **C** $7xy$

   **D** $6x - y$

   Answer at the back of the book.

4. Simplify $7x + 4 - 3x + 1$.

   **A** $4x + 3$

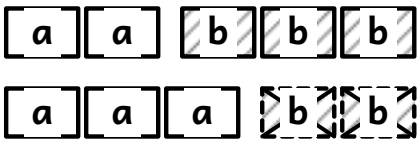
   **B** $10x + 5$

   **C** $9x$

   **D** $4x + 5$

   Answer at the back of the book.

5. The first row is $2a + 3b$. The second row is $3a - 2b$, and the crossed bars are taken away. What is the whole picture, simplified?



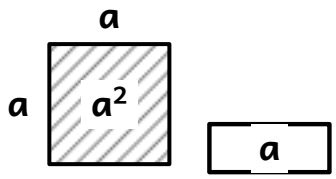

   **A** $5a + 5b$

   **B** $5ab$

   **C** $5a + b$

   Answer at the back of the book.

6. Look at the picture. Which sentence is true about $a$ and $a^2$?




**A** They are like terms, because both have the letter $a$.

**B** They are not like terms. One is a bar and the other is a square.

**C** They are equal.

Answer at the back of the book.

7. Collect the like terms in $3a + 2b + 5a$. What is the number in front of $a$?

Answer at the back of the book.

8. In $2b + 6b - 3b$, how many bars of b are left after collecting?

Answer at the back of the book.

9. **From an earlier chapter**

Find the value of $3a + 2b$ when $a = 4$ and $b = 5$.

**A** 14

**B** 59

**C** 22

Answer at the back of the book.

**KEEP IT**

Copy this into your exercise book.

**Like terms have the same letter part. Collect them: $3a + 5a = 8a$. A letter alone, such as b, means 1b. Different letters stay apart.**

**Draw:** Draw bars for $3a + 2b + 5a - b$. Group the $a$ bars and the b bars. Write $8a + b$.

M3.8.4 CHAPTER 4

# Brackets

> **Each of 3 children gets a tin with x beans and 4 more loose beans. How many beans is that in all?**

---

**YOUR GUESS**

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 3x + 12. Each child gets x beans and 4 beans, so there are 3 lots of both.
- ☐ **B** 3x + 4. Only the tins are tripled, and the 4 stays the same.
- ☐ **C** 7x. Add the 3 and the 4, and keep the x.

**IF THIS FEELS HARD, READ THIS FIRST**

- Level 3, Long multiplication and division, M3.2.6. Brackets are worked out first. If that is new, read this first.
- Level 3, Negative numbers, factors and powers, M3.1.3. You need to find the factors of two numbers. Read this if you are not sure how.

**BY THE END YOU CAN**

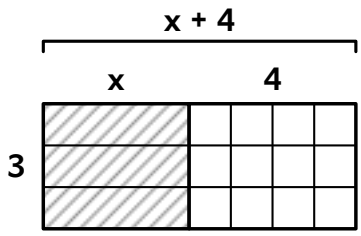
- Expand 3(x + 4) to get 3x + 12
- Factorise 6x + 9 as 3(2x + 3)

Each of 3 children gets a tin with x beans and 4 loose beans. One child has x + 4 beans. The 3 children have 3(x + 4) beans.

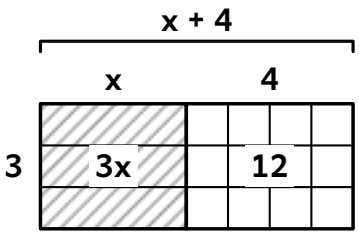
The signs ( ) are **brackets**. The 3 outside the brackets multiplies everything inside them. A rectangle shows what that means.

DRAW IT FIRST

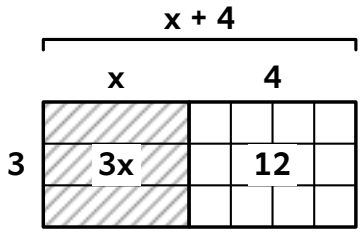
## Draw 3(x + 4) as a rectangle




**1** Draw a rectangle 3 high and x + 4 across.




**2** Cut it into two parts. The x part is 3 bars, so it is 3x. The 4 part has 3 × 4 = 12 squares.




**3(x + 4) = 3x + 12**

**3** The whole rectangle is both parts: 3(x + 4) = 3x + 12.

To **expand** the brackets, we multiply every term inside by the number outside. Both terms are multiplied, not just the first.

WORKED EXAMPLE

**Expand 3(x + 4).**

1. Multiply the first term: 3 × x = 3x.
2. Multiply the second term: 3 × 4 = 12.
3. Put them together: 3(x + 4) = 3x + 12.

**Answer:** 3x + 12

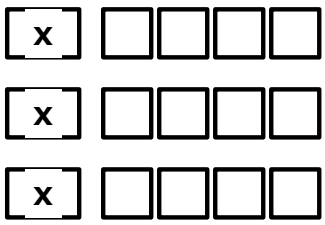
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SEE IT THREE WAYS

**With things**

Make 3 rows. In each row put one cup of x beans and 4 loose beans. Count the cups. Count the loose beans. The length of *a* bar does not tell you x.

**As *a* picture**




3(x + 4)

**In symbols**

**3(x + 4) = 3x + 12.**

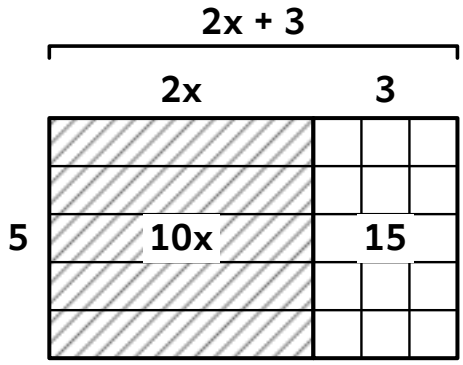
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WORKED EXAMPLE

**Expand 5(2x + 3).**

1. Multiply the first term: 5 × 2x = 10x.
2. Multiply the second term: 5 × 3 = 15.
3. Put them together: 5(2x + 3) = 10x + 15.

**Answer:** 10x + 15



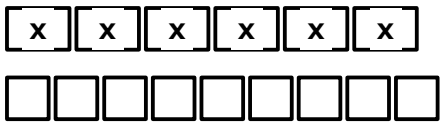

5(2x + 3) = 10x + 15. The 5 multiplies both parts.

Now go the other way. Start with 6x + 9. Both 6 and 9 can be divided by 3. A number that divides every term is a **common factor**.

To **factorise** is to take the common factor outside brackets. It undoes expanding. Take out the highest common factor. Then the numbers left inside the bracket have no common factor except 1.

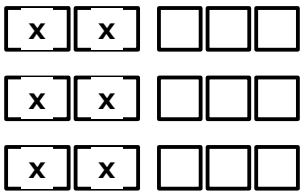
DRAW IT FIRST

## Share 6x + 9 into equal rows

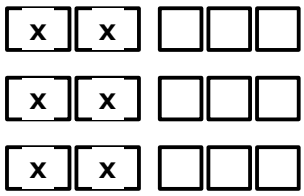


**6x + 9**

**1** Draw 6 bars of x and 9 squares. This is 6x + 9.

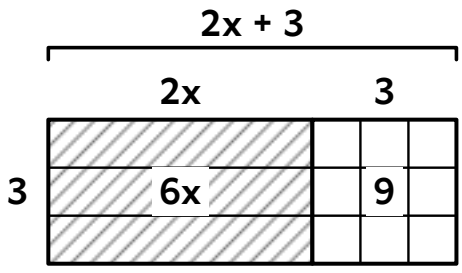


**2** Share them into 3 equal rows. Each row has 2 bars of x and 3 squares.



**6x + 9 = 3(2x + 3)**

**3** One row is 2x + 3. There are 3 rows, so 6x + 9 = 3(2x + 3).




The area is 6x + 9. One side is 3, so the other side is 2x + 3.

WORKED EXAMPLE

**Factorise 6x + 9.**

1. The highest common factor of 6 and 9 is 3.
2. Divide each term by 3: 6x ÷ 3 = 2x, and 9 ÷ 3 = 3.
3. Write 3 outside the bracket: 6x + 9 = 3(2x + 3).
4. Check by expanding: 3 × 2x + 3 × 3 = 6x + 9.

**Answer:** 3(2x + 3)

TRY IT

## Three rows on the table

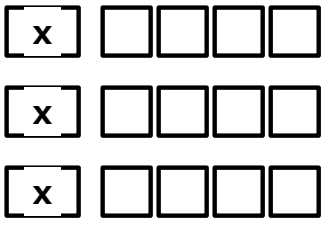
**You need:** 3 small cups with the same number of beans in each, 12 loose beans, paper and a pencil.

1. Make 3 rows. In each row put 1 cup and 4 loose beans.
2. Count the cups. There are 3. Count the loose beans: 3 × 4 = 12.
3. Write what is on the table: 3x + 12. Here x is the beans in one cup.
4. Now write one row as x + 4, and all 3 rows as 3(x + 4). Both say the same.

**What you will see:** The rows can be counted by row, 3(x + 4), or by kind, 3x + 12. Either way you count the same beans.

---

If you cannot try it, this is what happens.




3(x + 4) = 3x + 12

A WRONG IDEA MANY PEOPLE HAVE

~~**3(x + 4) = 3x + 4.**~~

**Why it feels right:** The 3 is next to the x, so it seems to multiply only the x.

**The test that breaks it:** Let x be 1. Then 3(x + 4) = 3 × 5 = 15. But 3x + 4 = 3 + 4 = 7. The answer 3x + 12 gives 3 + 12 = 15.

**What is true:** The number outside the bracket multiplies every term inside it.

KEY WORDS

**brackets**
the signs ( ) that group terms. A number outside multiplies all of them.

**expand**
to multiply out brackets, such as 3(x + 4) = 3x + 12.

**factorise**
to take a common factor outside brackets, such as 6x + 9 = 3(2x + 3).

**common factor**
a number that divides every term.

BACK TO YOUR GUESS

**The answer is A: 3x + 12. Each child gets x beans and 4 beans, so there are 3 lots of both.**

Every child has x + 4 beans, and there are 3 children. That is 3 lots of x and 3 lots of 4. So 3(x + 4) = 3x + 12.

---

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Expand $2(x + 5)$.

   **A** $2x + 5$

   **B** $x + 10$

   **C** $7x$

   **D** $2x + 10$

   Answer at the back of the book.

2. Expand $4(3n + 2)$.

   **A** $12n + 2$

   **B** $7n + 6$

   **C** $12n + 8$

   **D** $3n + 8$

   Answer at the back of the book.

3. Factorise $8x + 12$ by taking out the highest common factor.

   **A** $2(4x + 6)$

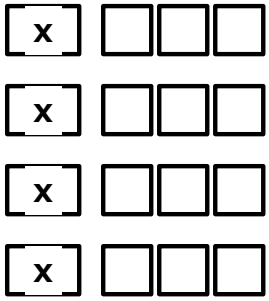
   **B** $4(2x + 3)$

   **C** $4(2x + 12)$

   **D** $8(x + 12)$

   Answer at the back of the book.

4. The picture has 4 rows. Each row has one bar of x and 3 small squares. Write the whole picture as one expression with a bracket.




**A** 4(x + 3)

**B** 4x + 3

**C** x + 12

**D** 4 + x + 3

Answer at the back of the book.

5. Mei expands 3(x + 4) and gets 3x + 4. Test her answer by putting in x = 1. What do you find?

**A** The values are 7 and 7, so she is right.

**B** The values are 15 and 15, so she is right.

**C** The values are 15 and 7, so she is wrong.

Answer at the back of the book.

6. **From an earlier chapter**

Simplify 3x + 4 + 2x − 1.

**A** 5x + 5

**B** 6x

**C** 9x

**D** 5x + 3

Answer at the back of the book.

7. Expand 4(x + 5). What is the plain number in your answer?

Answer at the back of the book.

8. You factorise 10x + 15 by taking out the highest common factor. What number goes outside the bracket?

   Answer at the back of the book.

**KEEP IT**

Copy this into your exercise book.

**The number outside brackets multiplies every term inside. 3(x + 4) = 3x + 12. To factorise, take out the highest common factor. Check by expanding.**

**Draw:** Draw a rectangle 3 high and x + 4 across. Cut it into 3x and 12, and write 3(x + 4) = 3x + 12.

M3.8.5 CHAPTER 5

# Solving equations in one step

> **A balance is level. The left pan holds a bag of x stones and 7 loose stones. The right pan holds 12 loose stones. How many stones are in the bag?**

---

**YOUR GUESS**

Pick one before you read on. It is fine to be wrong.

- ☐ **A** x = 5. Take 7 stones from both sides, and the bag balances 5 stones.
- ☐ **B** x = 19. Add the 7 and the 12.
- ☐ **C** x = 12. The right pan shows the answer.

**IF THIS FEELS HARD, READ THIS FIRST**

- Level 2, Patterns and rules, M2.8.4. Finding a missing number with the opposite operation is the same idea. Read this if it is new.

**BY THE END YOU CAN**

- Use a balance picture to keep both sides equal
- Solve x + 7 = 12 and 4x = 20

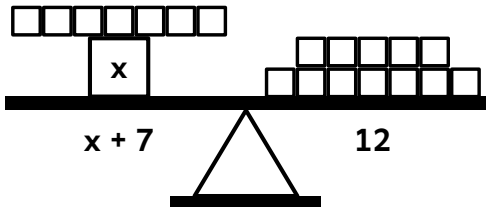
A balance is level when both sides weigh the same. Every stone weighs the same. A bag holds x stones, and we do not know x yet. The bag itself weighs nothing.

The left pan holds the bag and 7 loose stones, so it weighs x + 7. The right pan holds 12 loose stones. We write x + 7 = 12.

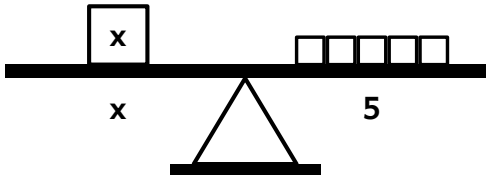
This is an **equation**. It says that two things are equal. To **solve** it is to find the number that x stands for.

DRAW IT FIRST

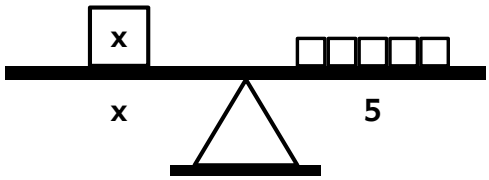
## Take the same from both sides




**1** The balance is level. The left side is x + 7 and the right side is 12.




**2** Take 7 stones from each side. The balance stays level, and the bag is on its own.




**x = 5**

**3** The bag balances 5 stones, so x = 5.

Here is the rule. Whatever you do to one side, do to the other side. Then the two sides stay equal.

**WORKED EXAMPLE**

**Solve x + 7 = 12.**

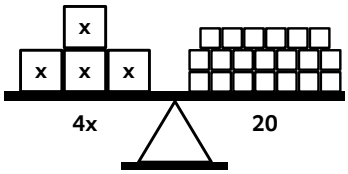
1. The 7 is added to x. The opposite of adding 7 is taking away 7.
2. Take 7 from both sides: x = 12 − 7 = 5.
3. So x = 5.
4. Check: put 5 back into x + 7. We get 5 + 7 = 12. It is true.

**Answer:** x = 5

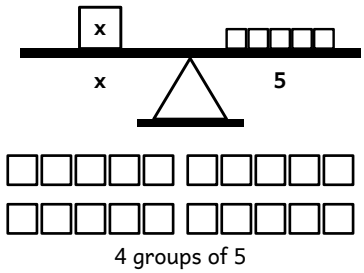
Adding and taking away undo each other. Multiplying and dividing undo each other. Each one is the **inverse operation** of the other.

DRAW IT FIRST

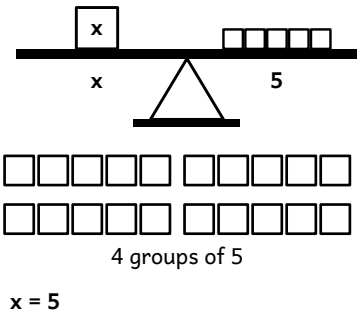
## Share both sides into 4 equal groups




**1** Four bags balance 20 stones. This is 4x = 20.




**2** Share each side into 4 equal groups. The 20 stones make 4 groups of 5. One bag balances 5 stones.




**x = 5**

**3** So x = 5.

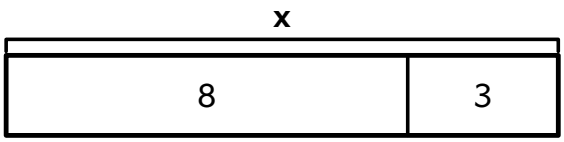
WORKED EXAMPLE

**Solve 4x = 20.**

1. 4x means 4 × x. The opposite of multiplying by 4 is dividing by 4.
2. Divide both sides by 4: 4x ÷ 4 = 20 ÷ 4.
3. Work out 20 ÷ 4 = 5. So x = 5.
4. Check: put 5 back into 4x. We get 4 × 5 = 20. It is true.

**Answer:** x = 5

What if something is taken away from x? Look at x − 3 = 8. The whole x is cut into two parts. One part is 3, and the other part is 8.




x − 3 = 8. Put the 3 back, and the whole bar is x.

WORKED EXAMPLE

**Solve x − 3 = 8.**

1. The 3 is taken away from x. The opposite of taking away 3 is adding 3.
2. Add 3 to both sides of x − 3 = 8.
3. On the right side: 8 + 3 = 11.
4. So x = 11.
5. Check: put 11 back into x − 3. We get 11 − 3 = 8. It is true.

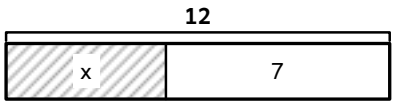
**Answer:** x = 11

SEE IT THREE WAYS

### With things

Count out 12 stones. Hide 7 of them in your hand. Count the stones left on the table.

### As a picture




### In symbols

**x + 7 = 12, so x = 12 − 7 = 5.**

TRY IT

## Make your own balance

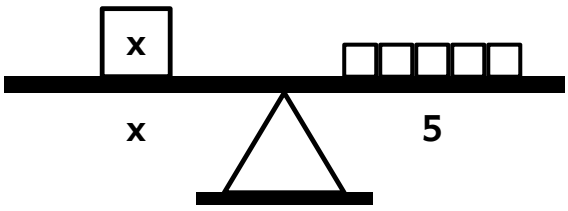
**You need:** a ruler or a flat stick, a pencil or a round bottle to rest it on, 2 cups that are the same, about 50 small stones or washers, all the same size.

1. Rest the stick across the pencil so that it can tip. Ask a friend to hide some stones in one cup.
2. Put that cup and 7 loose stones on the left. Put the empty cup on the right, so the two cups cancel. Add loose stones to the right until the stick is level.
3. Take 7 stones from the left. Take 7 stones from the right. Is the stick still level?
4. Now the stones in the cup balance the stones on the right. Count them. That is x.

**What you will see:** The two cups cancel. The stick stays level because you take the same from both sides. The stones left on the right tell you how many were in the cup.

---

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

**~~x + 7 = 12, so x = 12 + 7 = 19.~~**

**Why it feels right:** There is a 7 and a 12 and a plus sign, and you are used to adding.

**The test that breaks it:** Put 19 back into x + 7. You get 19 + 7 = 26, not 12. So 19 is not the answer.

**What is true:** Use the opposite operation. The 7 is added to x, so take 7 from both sides.

KEY WORDS

**equation**
two things that are equal, with a letter for an unknown number.

**solve**
to find the number that makes an equation true.

**inverse operation**
the operation that undoes another, such as taking away and adding.

BACK TO YOUR GUESS

**The answer is A: x = 5. Take 7 stones from both sides, and the bag balances 5 stones.**

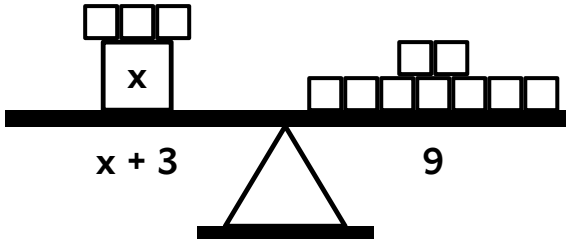
The pans stay level if we take the same from both sides. Take 7 stones from each pan. The bag is left on its own, and it balances 12 − 7 = 5 stones.

---

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. The balance is level. Solve x + 3 = 9.




**A** 12

**B** 6

**C** 3

Answer at the back of the book.

2. Solve 3x = 18.

**A** 15

**B** 21

**C** 54

**D** 6

Answer at the back of the book.

3. Solve x − 5 = 9.

**A** 4

**B** 9

**C** 14

Answer at the back of the book.

4. Solve x ÷ 4 = 3.

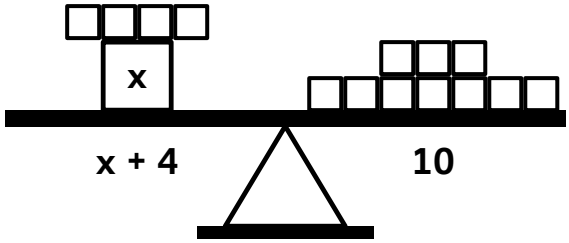
**A** 7

**B** 12

**C** 0.75

Answer at the back of the book.

5. The balance is level. Which move keeps it level?




**A** Take stones from the left side only.

**B** Take one whole bag from the left side only.

**C** Add stones to the left and take stones from the right.

**D** Take the same number of stones from both sides.

Answer at the back of the book.

6. Tariq solves x + 8 = 15 and says x = 23. Test his answer by putting it back into x + 8. What do you find?

**A** 23 + 8 = 31, not 15. His answer is wrong.

**B** 23 − 8 = 15, so he is right.

**C** 23 + 8 = 31, so he is right.

Answer at the back of the book.

7. **From an earlier chapter**

Expand 2(x + 3).

**A** 2x + 3

**B** 2x + 6

**C** x + 6

Answer at the back of the book.

8. Solve x + 9 = 20.

Answer at the back of the book.

9. Solve 6x = 42.

Answer at the back of the book.

**KEEP IT**

Copy this into your exercise book.

**An equation is a level balance. Do the same to both sides. Use the inverse operation: take away to undo add, divide to undo multiply. Always put the answer back.**

**Draw:** Draw a balance. Put x and 7 stones on the left and 12 stones on the right. Cross out 7 from each side. Write x = 5.

M3.8.6 CHAPTER 6

# Solving equations in two steps

> **A balance is level. The left pan holds 3 bags of x stones and 5 loose stones. The right pan holds 20 loose stones. How many stones are in each bag?**

---

**YOUR GUESS**

Pick one before you read on. It is fine to be wrong.

- ☐ **A** x = 5. Take 5 stones from each side, then share the 15 stones into 3 equal groups.
- ☐ **B** x = 15. Take 5 from 20 and stop, because 15 is what is left.
- ☐ **C** x = 8. Add the 3 and the 5.

**IF THIS FEELS HARD, READ THIS FIRST**

- Level 2, Patterns and rules, M2.8.3. Working back from an output to the input is the same idea. Read this if it is new.

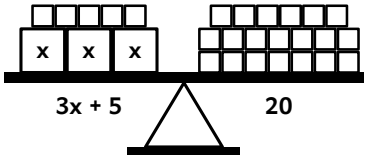
**BY THE END YOU CAN**

- Solve 3x + 5 = 20
- Solve equations with brackets, like 2(x + 3) = 14
- Solve 5x − 2 = 3x + 6, with x on both sides

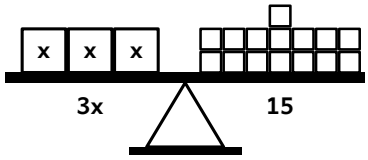
In the last chapter each equation took one step. Now the bag is multiplied and a number is added too. The equation 3x + 5 = 20 needs two steps. It is a **two-step equation**.

DRAW IT FIRST

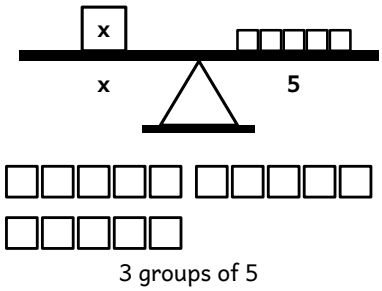
## Two steps on the balance



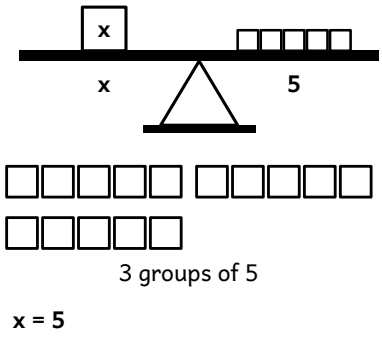

**1** Three bags and 5 stones balance 20 stones.




**2** Take 5 stones from each side. Three bags balance 15 stones.

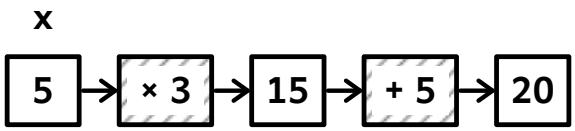



**3** Share each side into 3 equal groups. The 15 stones make 3 groups of 5. One bag balances 5 stones.

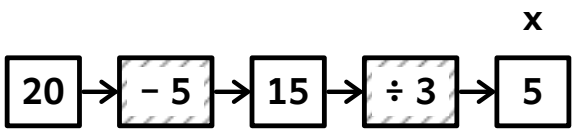



**4** So x = 5.

The order matters. In 3x + 5, x was first multiplied by 3, and then 5 was added. We undo the steps backwards. Take away 5 first, then divide by 3.




Building 3x + 5: multiply by 3, then add 5.




Solving it: start at 20, take away 5, then divide by 3.

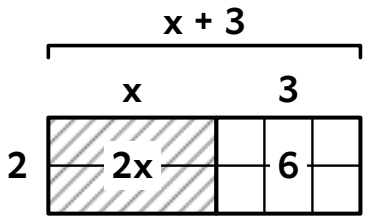
**WORKED EXAMPLE**

**Solve 3x + 5 = 20.**

1. Take 5 from both sides: 3x = 20 − 5 = 15.
2. Divide both sides by 3: x = 15 ÷ 3 = 5.
3. Check: put 5 back. 3 × 5 + 5 = 15 + 5 = 20. It is true.

**Answer:** x = 5

Some equations have brackets. In 2(x + 3) = 14, the 2 multiplies both x and 3. One way is to expand the bracket first.




2(x + 3) = 2x + 6. The whole rectangle has the area 14.

WORKED EXAMPLE

**Solve 2(x + 3) = 14.**

1. Expand the bracket: 2(x + 3) = 2x + 6, so 2x + 6 = 14.
2. Take 6 from both sides: 2x = 14 − 6 = 8.
3. Divide both sides by 2: x = 8 ÷ 2 = 4.
4. Check: put 4 back. 2 × (4 + 3) = 2 × 7 = 14. It is true.

**Answer:** x = 4

There is another way. Divide both sides by 2 first. This gives x + 3 = 7, and then x = 4. Both ways give the same answer.

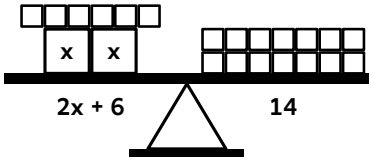
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SEE IT THREE WAYS

**With things**

Picture a see-saw. On one side are 2 bags of stones and 6 loose stones. On the other side are 14 loose stones.

**As a picture**




**In symbols**

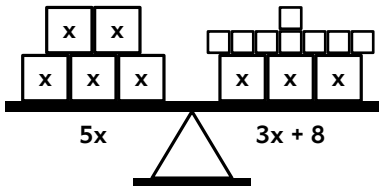
**2(x + 3) = 14 becomes 2x + 6 = 14. Then 2x = 8 and x = 4.**

---

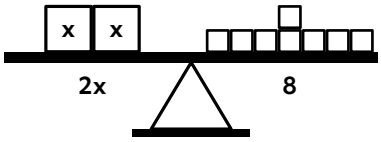
Sometimes x is on both sides. First take the same number of bags from both sides. This leaves the bags on one side only.

DRAW IT FIRST

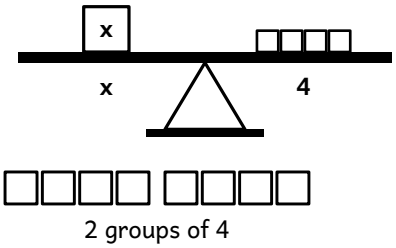
## Bags on both sides



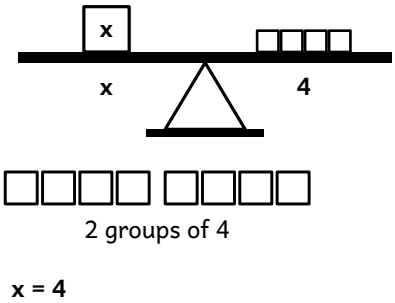

**1** Five bags balance 3 bags and 8 stones. This is 5x = 3x + 8.




**2** Take 3 bags from each side. Two bags balance 8 stones.




**3** Share each side into 2 equal groups. The 8 stones make 2 groups of 4. One bag balances 4 stones.




**4** So x = 4.

**WORKED EXAMPLE**

**Solve $5x - 2 = 3x + 6$.**

1. Add 2 to both sides: $5x = 3x + 6 + 2 = 3x + 8$.
2. Take $3x$ from both sides: $5x - 3x = 8$, so $2x = 8$.
3. Divide both sides by 2: $x = 8 \div 2 = 4$.
4. Check the left side: $5 \times 4 - 2 = 20 - 2 = 18$.
5. Check the right side: $3 \times 4 + 6 = 12 + 6 = 18$. Both sides match.

**Answer:** $x = 4$

TRY IT

## Two steps on *a* stick *b*alance

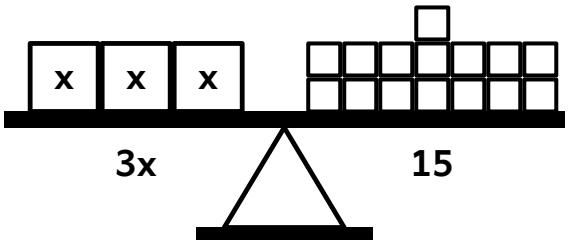
**You need:** *a* ruler or flat stick, *a* pencil to rest it on, 6 cups that *a*re the same, 50 small stones or washers, *a*ll the same size.

1. Ask *a* friend to put the same number of stones in each of 3 cups. Use 2 to 6 stones. Cover them. Rest the stick on the pencil.
2. Put the 3 full cups and 5 loose stones on the left. Put 3 empty cups on the right, so the cups cancel. Add loose stones to the right until the stick is level.
3. Take 5 stones from each side. Is the stick still level?
4. Share the stones on the right into 3 equal groups. One group is the number of stones in one cup.

**What you will see:** First you take *a*way the loose number. Then you share. The empty cups cancel the full ones, *a*nd the stick stays level *a*t every step.

---

If you cannot try it, this is what happens.




A WRONG IDEA MANY PEOPLE HAVE

**~~In 3x + 5 = 20, divide only the 3x by 3, *a*nd leave the rest.~~**

**Why it feels right:** The 3 is stuck to the x, so it seems the only thing to divide.

**The test that breaks it:** Try it. You get x + 5 = 20, so x = 15. Put 15 back: 3 × 15 + 5 = 50, not 20.

**What is true:** Take *a*way the 5 first. Or divide every term on both sides by 3.

KEY WORDS

**two-step equation**
an equation that needs two steps to solve, such as 3x + 5 = 20.

BACK TO YOUR GUESS

**The answer is A: x = 5. Take 5 stones from each side, then share the 15 stones into 3 equal groups.**

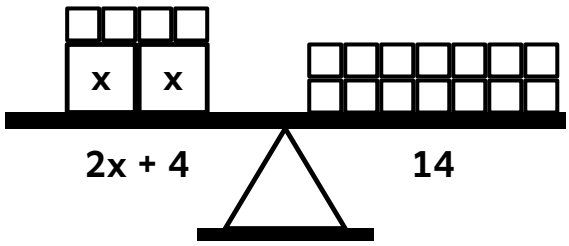
Two things happen to the bags: there are 3 of them, and 5 stones are added. Undo both. Take 5 from each side to get 3x = 15. Then divide by 3 to get x = 5.

---

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. The balance is level. Solve 2x + 4 = 14.




**A** 9

**B** 3

**C** 5

Answer at the back of the book.

2. Solve $4x - 3 = 17$.

   **A** 3.5

   **B** 5

   **C** 20

   Answer at the back of the book.

3. Solve $2(x + 3) = 14$.

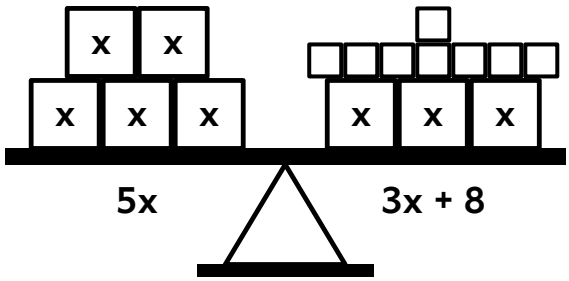
   **A** 4

   **B** 8

   **C** 11

   Answer at the back of the book.

4. The balance is level. The left side is 5 bags of x. The right side is 3 bags of x and 8 loose stones. Solve $5x = 3x + 8$.




   **A** 8

   **B** 1

   **C** 4

   Answer at the back of the book.

5. What is the right first step to solve $3x + 5 = 20$?

   **A** Divide only the $3x$ by 3.

   **B** Add the loose number to both sides.

   **C** Multiply both sides by the number of bags.

   **D** Take the loose number away from both sides.

   Answer at the back of the book.

6. Leila solves $2x + 1 = 11$ and says $x = 4$. Put 4 back into the equation. What do you find?

   **A** $2 \times 4 + 1 = 9$, so her answer is right.

   **B** $2 \times 4 + 1 = 9$, not 11. Her answer is wrong.

   **C** $4 + 1 = 5$, so her answer is wrong.

   Answer at the back of the book.

7. **From an earlier chapter**

   Solve $x - 6 = 10$.

   **A** 16

   **B** 4

   **C** 60

   Answer at the back of the book.

8. Solve $3x + 5 = 20$.

   Answer at the back of the book.

9. Solve $3(x + 2) = 24$.

   Answer at the back of the book.

**KEEP IT**

Copy this into your exercise book.

**Undo the steps backwards. Take away the loose number first, then divide. Expand a bracket first, or divide by the number outside. Always put the answer back.**

**Draw:** Draw a balance with 3 bags and 5 stones on the left and 20 stones on the right. Draw each step and write x = 5.

M3.8.7 CHAPTER 7

# Writing your own equations

> **Three pieces of rope of the same length and a 5 m piece are joined. The whole rope is 26 m long. How long is each of the 3 pieces?**

---

**YOUR GUESS**

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Each piece is 31 m. Add 5 to 26.
- ☐ **B** Each piece is 21 m. Take 5 from 26, and that is the answer.
- ☐ **C** Each piece is 7 m. The equation $3x + 5 = 26$ gives $x = 7$.

**BY THE END YOU CAN**

- Write an equation from a word problem and solve it
- Check an answer by putting it back into the equation

A **word problem** tells a story with numbers. To solve it, turn the story into an equation. Then solve the equation, and check the answer.

Words and symbols mean the same thing. This table shows how some words are written with a letter.

| words | symbols |
|---|---|
| add 5 to n | n + 5 |
| 3 times n | 3n |
| take 7 from n | n − 7 |
| double n, add 6 | 2n + 6 |
| is the same as | = |

Take care with the order: "take 7 from n" is n − 7, not 7 − n.

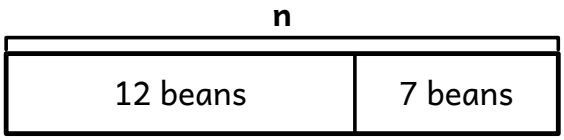
Here are four steps. First, pick a letter for the unknown, and say what it means. Second, write the equation. Third, solve it. Fourth, check the answer.

**WORKED EXAMPLE**

**Joseph has some beans. He gives 7 away and has 12 left. How many beans did he have at the start?**

1. Let n be the beans Joseph had at the start.
2. He gave away 7 and has 12 left, so n − 7 = 12.
3. Add 7 to both sides: n = 12 + 7 = 19.
4. Check: 19 − 7 = 12. It is true. He had 19 beans.

**Answer:** 19 beans



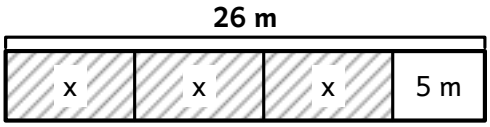

The whole bar is n. The 7 given away and the 12 left make n.

DRAW IT FIRST

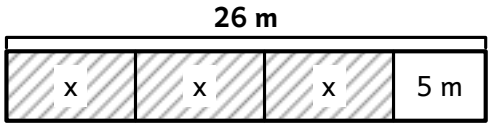
## From *a* story to *an* equation




**1** Let x be the length of one equal piece, in metres. Draw 3 pieces of x and the 5 m piece.

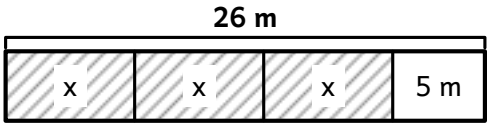



**2** The whole rope is 26 m. Write that over the bar.




**3x + 5 = 26**

**3** Write the equation from the bar: 3x + 5 = 26.




**3x + 5 = 26**

**x = (26 − 5) ÷ 3 = 7**

**4** Solve it. Take 5 from 26 and divide by 3. This gives x = 7.

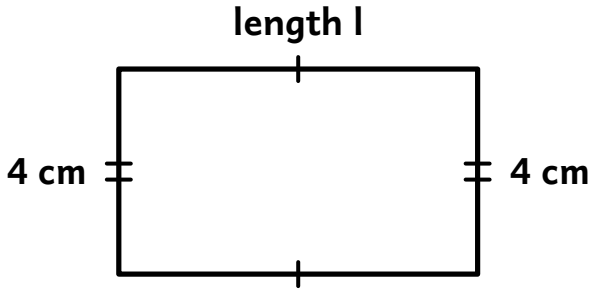
WORKED EXAMPLE

**Three pieces of rope of the same length and a 5 m piece are joined. The rope is 26 m long. How long is each of the 3 pieces?**

1. Let x be the length of one piece, in metres.
2. Three pieces and 5 m make 26 m, so 3x + 5 = 26.
3. Take 5 from both sides: 3x = 26 − 5 = 21.
4. Divide both sides by 3: x = 21 ÷ 3 = 7.
5. Check in the equation: 3 × 7 + 5 = 21 + 5 = 26. It is true.
6. Check in the story: three pieces of 7 m and one of 5 m make 26 m. Each piece is 7 m.

**Answer:** 7 m

Formulas are equations too. If you know the perimeter and one side of a rectangle, you can find the other side.




perimeter 22 cm

The perimeter is 22 cm. We find the length l.

WORKED EXAMPLE

**A rectangle is 4 cm wide. Its perimeter is 22 cm. How long is it?**

1. The formula is P = 2l + 2w. Put in P = 22 and w = 4: 22 = 2l + 2 × 4.
2. Work out the last part: 2 × 4 = 8.
3. So 2l + 8 = 22. Take 8 from both sides: 2l = 22 − 8 = 14.
4. Divide both sides by 2: l = 14 ÷ 2 = 7.
5. Check: 2 × 7 + 2 × 4 = 14 + 8 = 22. It is true.

**Answer:** 7 cm

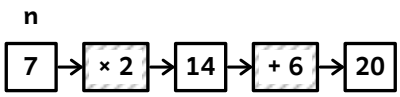
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SEE IT THREE WAYS

**With things**

Think of a number. Double it. Add 6. Tell a friend the answer, which is 20. Your friend must find your number.

**As a picture**




**In symbols**

**2n + 6 = 20, so 2n = 14 and n = 7.**

---

TRY IT

## Think of *a* number

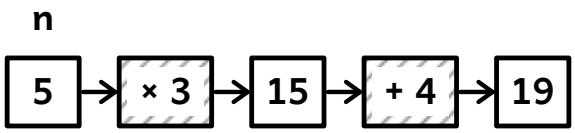
**You need:** *a* friend, paper *and a* pencil.

1. Think of *a* number, *and* write it down where your friend cannot see it.
2. Multiply it by 3, then *add* 4. Tell your friend only the *answer*.
3. Your friend writes the equation 3n + 4 = your *answer*, *and* solves it.
4. Show your number. Is it the same *as* the one your friend found?

**What you will see:** The equation *always* finds the number you thought of. If you thought of 5, the *answer* you told was 19, *and* 3n + 4 = 19 gives n = 5.

---

If you cannot try it, this is what happens.




A WRONG IDEA MANY PEOPLE HAVE

### ~~If my answer fits my equation, my answer must be right.~~

**Why it feels right:** The numbers match, so everything seems fine.

**The test that breaks it:** Suppose you write 3x − 5 = 16 for the rope story by mistake. You get x = 7. Check: 3 × 7 − 5 = 16. It fits. But 3 pieces of 7 m *and* 5 m make 26 m, not 16 m.

**What is true:** Check in the story *as* well *as* in the equation. A wrong equation *can* still fit its own *answer*.

KEY WORDS

**word problem**
*a* story with numbers, to be turned into *an* equation.

**solution**
the number that makes *an* equation true.

BACK TO YOUR GUESS

**The answer is C: Each piece is 7 m. The equation 3x + 5 = 26 gives x = 7.**

Let x be the length of one piece. Three pieces and 5 m make 26 m, so 3x + 5 = 26. Take 5 from both sides, then divide by 3. Each piece is 7 m.

---

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Joseph has n beans. He gets 4 more and then has 15. Which equation is right, and what is n?

   **A** n − 4 = 15, so n = 19

   **B** n + 4 = 15, so n = 11

   **C** 4n = 15, so n = 3.75

   Answer at the back of the book.

2. Amina thinks of a number. She doubles it and takes away 3. The answer is 11. Which equation is right, and what is her number?

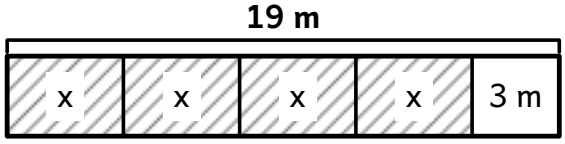
   **A** 2n − 3 = 11, so n = 7

   **B** n − 3 = 11, so n = 14

   **C** 2n + 3 = 11, so n = 4

   Answer at the back of the book.

3. The bar shows 4 equal pieces of rope and one piece of 3 m. The whole rope is 19 m. How long is each equal piece?




**A** 16 m

**B** 5.5 m

**C** 4 m

Answer at the back of the book.

4. Lucía says the solution of $5x + 2 = 32$ is $x = 7$. Put 7 back into the equation. What do you find?

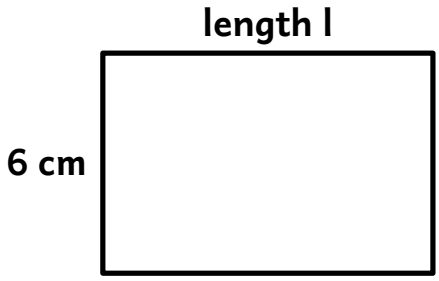
**A** $5 \times 7 + 2 = 37$, so she is right.

**B** $5 \times 7 + 2 = 37$, not 32. She is wrong.

**C** $5 + 7 + 2 = 14$, so she is wrong.

Answer at the back of the book.

5. A rectangle is 6 cm wide. Its perimeter is 30 cm. Which equation finds the length $l$?




**A** $l + 6 = 30$

**B** $2l + 6 = 30$

**C** $6l = 30$

**D** $2l + 12 = 30$

Answer at the back of the book.

6. **From an earlier chapter**

   Solve 3x + 4 = 19.

   **A** 5

   **B** 15

   **C** 23

   Answer at the back of the book.

7. I think of a number. I multiply it by 3 and add 4. The answer is 19. What is my number?

   Answer at the back of the book.

8. Joseph has some beans. He gives 7 away and has 12 left. How many beans did he have at the start?

   Answer at the back of the book.

**KEEP IT**

Copy this into your exercise book.

**Pick a letter and say what it means. Write the equation from the story. Solve it. Check in the equation and in the story.**

**Draw:** Draw a bar for the rope story. It has 3 parts of x and a part of 5. Write 3x + 5 = 26.

M3.8.8 CHAPTER 8

# Sequences and the nth term

> **A dot pattern has 5, 8, 11 and 14 dots in its first four pictures. How many dots will the 100th picture have?**

---

**YOUR GUESS**

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 500 dots, because the first picture has 5 dots and 5 × 100 = 500.
- ☐ **B** 103 dots, because each picture adds 3 to the picture number.
- ☐ **C** 302 dots, because each picture has 3 times its number, plus 2.

**IF THIS FEELS HARD, READ THIS FIRST**

- Level 2, Patterns and rules, M2.8.1. You need to continue a sequence that goes up in equal steps. Read this if it is new.
- Level 2, Patterns and rules, M2.8.7. Finding a far term without drawing every step is the idea of this chapter.

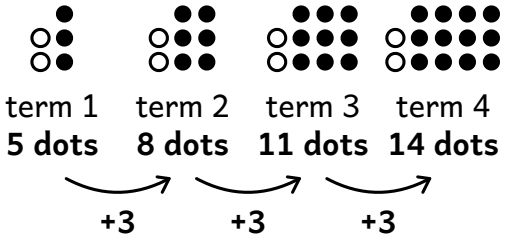
**BY THE END YOU CAN**

- Find the nth term of a sequence like 5, 8, 11, 14
- Use the nth term to find the 100th term

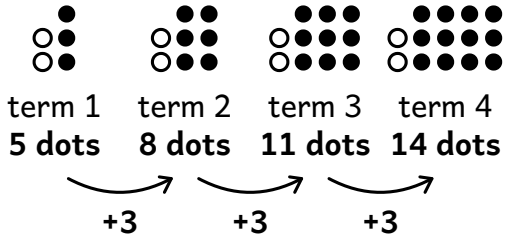
A **sequence** is a list of numbers that follows a rule. Each number in the list is a term. Here the sequence is 5, 8, 11, 14.

DRAW IT FIRST

## Count the dots in each picture

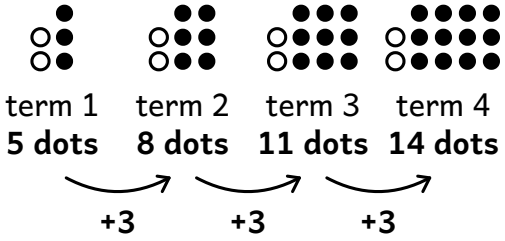



**1** Draw the pictures and count the dots: 5, 8, 11, 14.




black: 3 more. ring: always 2

**2** Each picture adds one column of 3 black dots. The 2 ring dots never change.




black: 3 more. ring: always 2

**dots = 3 × picture number + 2**

**3** Picture number n has 3 × n + 2 dots.

Each term has a **term number**. The first term has term number 1, the second has term number 2, and so on. We call the term number n.

The rule that gives any term from its term number is the **nth term**. For this sequence it is 3n + 2. We will see how to find it.

Look at how far each term is from the next. The terms go up by 3 each time. This is the **common difference**. It is the number in front of n.

| n | 3n | term |
|---|---|---|
| 1 | 3 | 5 |
| 2 | 6 | 8 |
| 3 | 9 | 11 |
| 4 | 12 | 14 |

Every term is 2 more than 3n.

WORKED EXAMPLE

**Find the nth term of 5, 8, 11, 14.**

1. The terms go up by 3 each time. The common difference is 3, so the nth term starts with 3n.
2. Write 3n for each term number: 3, 6, 9, 12.
3. Compare with the sequence: 5 − 3 = 2, 8 − 6 = 2, 11 − 9 = 2.
4. Each term is 2 more than 3n. So the nth term is 3n + 2.
5. Check with n = 4: 3 × 4 + 2 = 14. It matches the fourth term.

**Answer:** 3n + 2

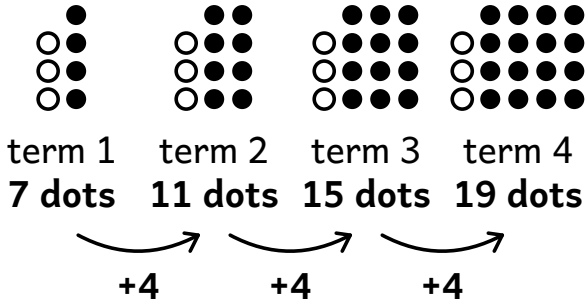
Now we can find any term. We do not need to draw all the pictures. We put the term number into the nth term.

WORKED EXAMPLE

**The nth term of *a* sequence is 3n + 2. Find the 100th term.**

1. The 100th term has n = 100.
2. Put n = 100 into 3n + 2. This gives 3 × 100 + 2.
3. Multiply first: 3 × 100 = 300.
4. Then add: 300 + 2 = 302.

**Answer:** 302




The sequence 7, 11, 15, 19 goes up by 4, so its nth term starts with 4n.

WORKED EXAMPLE

**Find the nth term of 7, 11, 15, 19. Then find the 50th term.**

1. The terms go up by 4 each time, so the nth term starts with 4n.
2. The first term of 4n is 4. The first term of the sequence is 7. And 7 − 4 = 3.
3. So the nth term is 4n + 3.
4. For n = 50: 4 × 50 = 200.
5. Then 200 + 3 = 203.

**Answer:** 4n + 3 and 203

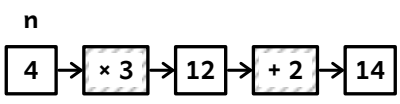
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SEE IT THREE WAYS

**With things**

Put 2 stones in a ring. Add a column of 3 stones, and count all of them. Add another column, and count again.

**As a picture**




**In symbols**

**The nth term is 3n + 2. With n = 4 it is 3 × 4 + 2 = 14.**

---

TRY IT

## Grow a pattern with stones

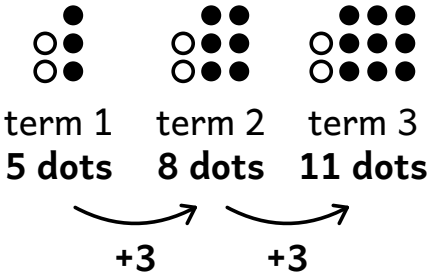
**You need:** about 40 small stones or beans.

1. Put 2 stones in a ring. Put a column of 3 stones beside it. Count them all.
2. Add another column of 3 stones. Count them all again.
3. Add one more column of 3. Write the three counts.
4. Guess how many stones there are with 10 columns. Then check with 3 × 10 + 2.

**What you will see:** The counts are 5, 8 and 11. With 10 columns there are 3 × 10 + 2 = 32 stones. The ring of 2 is the "plus 2" in the rule.

---

If you cannot try it, this is what happens.




A WRONG IDEA MANY PEOPLE HAVE

**~~The nth term of 5, 8, 11, 14 is n + 3, because the terms go up by 3.~~**

**Why it feels right:** The sequence adds 3 each time, so adding 3 seems to be the rule.

**The test that breaks it:** Put in n = 1. Then n + 3 is 4, but the first term is 5. Put in n = 4. Then n + 3 is 7, but the fourth term is 14.

**What is true:** The common difference is the number in front of n: it is 3n. Then fix the start: 3n + 2.

KEY WORDS

**sequence**
*a* list of numbers that follows *a* rule.

**term number**
the place of *a* term in the list: 1 for the first, 2 for the second.

**nth term**
*a* rule that gives *any* term from its term number, such *as* 3n + 2.

**common difference**
the *amount a* sequence goes up by each time.

BACK TO YOUR GUESS

**The answer is C: 302 dots, because each picture has 3 times its number, plus 2.**

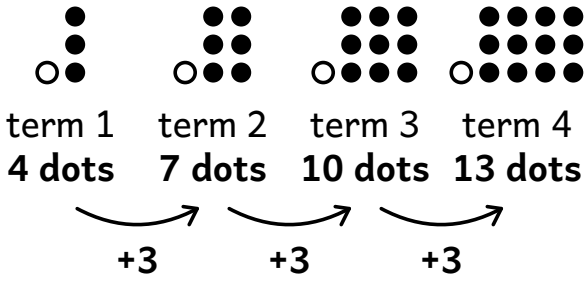
Each new picture adds *a* column of 3 dots, *and* 2 ring dots never change. Picture number n has 3n + 2 dots. For n = 100 that is 3 × 100 + 2 = 302.

---

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. The dot pictures have 4, 7, 10 and 13 dots. What is the nth term?




**A** n + 3

**B** 4n

**C** 3n

**D** 3n + 1

Answer *at* the back of the book.

2. The nth term of *a* sequence is 3n + 2. What is the 50th term?

   **A** 152

   **B** 352

   **C** 55

   Answer *a*t the back of the book.

3. Is 62 *a* term of the sequence 5, 8, 11, 14, ...?

   **A** No. 62 is not in the list 5, 8, 11, 14.

   **B** Yes. 3n + 2 = 62 gives n = 20, *a* whole number.

   **C** No. 62 cannot be divided by 3.

   Answer *a*t the back of the book.

4. Find the nth term of 7, 11, 15, 19, ...

   **A** 4n + 7

   **B** n + 4

   **C** 4n + 3

   Answer *a*t the back of the book.

5. What is the common difference of the sequence 5, 8, 11, 14?

   **A** 3

   **B** 2

   **C** 5

   Answer *a*t the back of the book.

6. Joseph says the nth term of 5, 8, 11, 14 is n + 3. Test it with n = 2. What do you find?

   **A** n + 3 gives 5 for n = 2, so he is right.

   **B** n + 3 gives 5 for n = 2, but the second term is 8. He is wrong.

   **C** n + 3 gives 8 for n = 2, so he is right.

   Answer *a*t the back of the book.

7. **From an earlier chapter**

   Solve $2x + 7 = 19$.

   **A** 12

   **B** 13

   **C** 6

   Answer at the back of the book.

8. The nth term of a sequence is $4n + 3$. What is the 20th term?

   Answer at the back of the book.

9. What is the common difference of the sequence 6, 11, 16, 21?

   Answer at the back of the book.

**KEEP IT**

Copy this into your exercise book.

**Find the common difference. It is the number in front of n. Compare with that many n, and add what is left. Then put in n = 100 to jump far ahead.**

**Draw:** Write 5, 8, 11, 14 and under it 3, 6, 9, 12. Write 2 more under each. Write the nth term $3n + 2$.

# Key words, A to Z

Every key word in this book, what it means, and the chapter that teaches it.

**brackets**
the signs ( ) that group terms. A number outside multiplies all of them. M3.8.4

**coefficient**
the number in front of a letter. In 3n it is 3. M3.8.1

**common difference**
the amount a sequence goes up by each time. M3.8.8

**common factor**
a number that divides every term. M3.8.4

**equation**
two things that are equal, with a letter for an unknown number. M3.8.5

**expand**
to multiply out brackets, such as 3(x + 4) = 3x + 12. M3.8.4

**expression**
numbers, letters and signs written together, such as n + 5. M3.8.1

**factorise**
to take a common factor outside brackets, such as 6x + 9 = 3(2x + 3). M3.8.4

**formula**
a rule written with letters, such as P = 2l + 2w. M3.8.2

**inverse operation**
the operation that undoes another, such as taking away and adding. M3.8.5

**like terms**
terms with the same letter part, such as 3a and 5a. M3.8.3

**nth term**
a rule that gives any term from its term number, such as 3n + 2. M3.8.8

**sequence**
a list of numbers that follows a rule. M3.8.8

**simplify**
to write an expression in its shortest form. M3.8.3

**solution**
the number that makes an equation true. M3.8.7

**solve**
to find the number that makes an equation true. M3.8.5

**squared**
multiplied by itself. n squared is n × n, written $n^2$. M3.8.1

**substitute**
to put a number in place of a letter. M3.8.2

**term**
one part of an expression, with the sign in front of it. M3.8.3

**term number**
the place of a term in the list: 1 for the first, 2 for the second. M3.8.8

**two-step equation**
an equation that needs two steps to solve, such as 3x + 5 = 20. M3.8.6

**unknown**
a number we do not know yet, written as a letter. M3.8.1

**value**
the number an expression gives when you substitute. M3.8.2

**word problem**
a story with numbers, to be turned into an equation. M3.8.7

# Answers

## M3.8.1. Letters for unknown numbers

1. **n + 4**

   The box is n and the loose beans are 4 more. Adding gives n + 4.

   - **If you chose B:** This says 4 boxes. Ravi has one box and 4 loose beans. Go to M3.8.1.
   - **If you chose C:** We write a number before a letter, not after it. A plus sign must also join the box and the beans. Go to M3.8.1.
   - **If you chose D:** A minus sign takes beans away. Ravi has 4 beans more, not fewer. Go to M3.8.1.

2. **3n**

   3 boxes of n beans is n + n + n. We write this as 3 × n, or 3n.

   - **If you chose A:** This is one box and 3 more beans. Tariq has 3 boxes. Go to M3.8.1.
   - **If you chose B:** We put the number first and the letter after it. n3 is not written. Go to M3.8.1.
   - **If you chose D:** This is 3 loose beans and one box. Tariq has 3 boxes, which is 3 times n. Go to M3.8.1.

3. **n × n**

   $n^2$ is n multiplied by itself. The area of a square with side n is n × n.

   - **If you chose A:** The small 2 does not say to multiply by 2. If n is 5, then n × 2 is 10, but $n^2$ is 25. Go to M3.8.1.
   - **If you chose B:** This is 2n, two boxes of n. The square has n rows of n. Go to M3.8.1.
   - **If you chose C:** The small 2 is not added. It says n is multiplied by itself. Go to M3.8.1.

4. **2n is 10 and $n^2$ is 25**

   2n is 2 × 5 = 10. $n^2$ is 5 × 5 = 25. They are not the same, so Mei is wrong.

   - **If you chose A:** You worked $n^2$ as 2 × 5. But $n^2$ is 5 × 5. Go to M3.1.7.
   - **If you chose C:** These are the wrong way round. 2n is 5 + 5, which is small. $n^2$ is 5 × 5. Go to M3.8.1.
   - **If you chose D:** You worked 2n as 5 × 5. But 2n is 2 × 5, which is 10. Go to M3.8.1.

5. **n − 3**

   She starts with n and takes 3 away, so she has n − 3 left.

   - **If you chose B:** Adding 3 gives her more stickers. She gives 3 away, so she has fewer. Go to M3.8.1.
   - **If you chose C:** This takes n away from 3. Amina takes 3 away from n. The order matters. Go to M3.8.1.
   - **If you chose D:** 3n means 3 boxes of n. Nobody is multiplying here. Amina gives away 3. Go to M3.8.1.

6. **2n + 3**

   There are 2 bars of n, which is 2n. The 3 squares are 3 more. Together they are 2n + 3.

   - **If you chose A:** You added the 2 and the 3. But the 3 squares are loose beans, not boxes. Only the bars are n. Go to M3.8.1.
   - **If you chose B:** There are 2 bars, so 2 boxes, not 1. The 2 boxes make 2n. Go to M3.8.1.
   - **If you chose D:** This is 2 loose beans and 3 boxes. The picture has 2 boxes and 3 loose beans. Go to M3.8.1.

7. **14 beans**

   n + 5 with n = 9 is 9 + 5 = 14.

8. **36**

   $n^2$ is n × n. With n = 6 it is 6 × 6 = 36.

## M3.8.2. Putting numbers in

1. **17**

   3n is 3 × 5 = 15. Then 15 + 2 = 17.

   - **If you chose A:** You put 3 and 5 side by side as 35. But 3n means 3 × 5 = 15. Go to M3.8.2.
   - **If you chose B:** You added 5 and 2 first. Multiply before you add: 3 × 5 = 15, then 15 + 2. Go to M3.2.6.
   - **If you chose C:** You added 3 + 5 + 2. A number next to a letter means multiply: 3 × 5. Go to M3.8.2.

2. **12**

   2n is 2 × 4 = 8. Then 20 − 8 = 12.

   - **If you chose B:** You left out the 2. The term 2n is 2 × 4 = 8, not 4. Go to M3.8.2.
   - **If you chose C:** You did 20 − 2 first and then multiplied by 4. Multiply first: 2 × 4 = 8. Go to M3.2.6.

3. **26 cm**

   2 × 9 = 18 and 2 × 4 = 8. Then 18 + 8 = 26 cm.

   - **If you chose A:** You multiplied 9 × 4. That is the area inside, not the distance round the edge. Go to M3.8.2.
   - **If you chose B:** You added one length and one width. Going round, you pass 2 lengths and 2 widths. Go to M3.8.2.
   - **If you chose D:** You doubled the length but not the width. Both sides come twice. Go to M3.8.2.

4. **2 × 7 + 2 × 3 = 20**

   2l is 2 × 7 = 14 and 2w is 2 × 3 = 6. Then 14 + 6 = 20.

   - **If you chose A:** 2l means 2 times l, not 2 and 7 side by side. Go to M3.8.2.
   - **If you chose B:** This is only half the way round. Multiply each side by 2. Go to M3.8.2.
   - **If you chose C:** The formula has a plus sign. It adds the two lengths and the two widths. Go to M3.8.2.

5. **n + 4 is bigger. It is 7, and 2n is 6.**

   With n = 3, n + 4 is 3 + 4 = 7. And 2n is 2 × 3 = 6. So n + 4 is bigger.

   - **If you chose A:** Not always. With n = 3, 2n is 6 but n + 4 is 7. Put the number in and check. Go to M3.8.2.
   - **If you chose C:** Put in n = 3. Then 2n is 6 and n + 4 is 7. They are not equal. Go to M3.8.2.

6. **5n + 2**

   5 bars of n make 5n. The 2 loose squares are 2 more. Together they are 5n + 2.

   - **If you chose A:** There are 5 bars, so 5 boxes of n. You cannot add the 5 and the 2 together. Go to M3.8.1.
   - **If you chose B:** This is 5 loose beans and 2 boxes. The picture has 5 boxes and 2 loose beans. Go to M3.8.1.
   - **If you chose D:** A number in front of a letter says how many of it. It is not a two digit number. Go to M3.8.1.

7. **25**

   4n is 4 × 6 = 24. Then 24 + 1 = 25.

8. **38 cm**

   2 × 12 = 24 and 2 × 7 = 14. Then 24 + 14 = 38 cm.

## M3.8.3. Collecting like terms

1. **4x and 7x**

   Both terms have the letter x and nothing else. The numbers in front may differ.

   - **If you chose A:** The letters are different. 4x is a number of x, and 7y is a number of y. Go to M3.8.3.
   - **If you chose C:** The 7 is a plain number. It has no x, so it is not like 4x. Go to M3.8.3.
   - **If you chose D:** $x^2$ is a square, but x is a bar. They are different shapes, so they are not like terms. Go to M3.8.3.

2. **8*a***

   5 bars of *a* and 3 bars of *a* make 8 bars of *a*. So 5*a* + 3*a* = 8*a*.

   - **If you chose B:** You multiplied 5 × 3. When you collect like terms, you add the numbers in front. Go to M3.8.3.
   - **If you chose C:** The letter stays. 5 bars of *a* and 3 more bars of *a* are 8 bars of *a*, not just 8. Go to M3.8.3.
   - **If you chose D:** *a* + *a* is 2*a*, not *a* square. When we add like terms, the letter part does not change. Go to M3.8.1.

3. **6x + y**

   The x terms: 4x + 2x = 6x. The y terms: 3y − 2y = y. So the answer is 6x + y.

   - **If you chose A:** You added 3y and 2y. But the sign in front of 2y is minus, so it is taken away. Go to M3.8.3.
   - **If you chose C:** You joined everything into one term. The x terms and the y terms stay apart. Go to M3.8.3.
   - **If you chose D:** You took the 3y away from the 2y. But we start with 3y, then take 2y away. That leaves 1y. Go to M3.8.3.

4. **4x + 5**

   The x terms: 7x − 3x = 4x. The plain numbers: 4 + 1 = 5. So the answer is 4x + 5.

   - **If you chose A:** The last term is plus 1, not minus 1. The plain numbers are 4 and 1, which make 5. Go to M3.8.3.
   - **If you chose B:** The sign in front of 3x is minus. So 7x − 3x is 4x. Go to M3.8.3.
   - **If you chose C:** You joined the plain numbers to the x terms. A plain number is not like *a* term with x. Go to M3.8.3.

5. **5*a* + b**

   The *a* bars: 2 + 3 = 5. The b bars: 3 − 2 = 1. So the answer is 5*a* + b.

   - **If you chose A:** You counted the crossed bars of b. They are taken away, so they cancel 2 of the 3 bars. Go to M3.8.3.
   - **If you chose B:** You joined the letters. Bars of *a* and bars of b are different, so they stay apart. Go to M3.8.3.

6. **They are not like terms. One is *a* bar and the other is *a* square.**

   Like terms have the same letter part. The terms $a$ and $a^2$ have different letter parts. They cannot join into one term.

   - **If you chose A:** $a^2$ is $a$ times $a$, $a$ square. The term $a$ is $a$ bar. The shapes differ, so they are not like terms. Go to M3.8.3.
   - **If you chose C:** They are different shapes. If $a$ is 3, then $a$ is 3 but $a^2$ is 9. Go to M3.8.1.

7. **8**

   The $a$ terms are $3a$ and $5a$. $3 + 5 = 8$, so the answer is $8a + 2b$.

8. **5**

   $2 + 6 = 8$, and $8 - 3 = 5$. So $2b + 6b - 3b = 5b$.

9. **22**

   $3a$ is $3 \times 4 = 12$ and $2b$ is $2 \times 5 = 10$. Then $12 + 10 = 22$.

   - **If you chose A:** You added every number: $3 + 4 + 2 + 5$. A number next to $a$ letter means multiply. Go to M3.8.2.
   - **If you chose B:** You put the numbers side by side: 34 and 25. $3a$ means $3 \times 4$, and $2b$ means $2 \times 5$. Go to M3.8.2.

## M3.8.4. Brackets

1. **2x + 10**

   $2 \times x = 2x$ and $2 \times 5 = 10$. So $2(x + 5) = 2x + 10$.

   - **If you chose A:** You multiplied only the first term. The 2 multiplies the 5 as well. Go to M3.8.4.
   - **If you chose B:** You multiplied only the second term. The 2 multiplies the x as well. Go to M3.8.4.
   - **If you chose C:** You added 2 and 5 and joined them to x. The 2 multiplies each term inside the bracket. Go to M3.8.4.

2. **12n + 8**

   4 × 3n = 12n and 4 × 2 = 8. So 4(3n + 2) = 12n + 8.

   - **If you chose A:** You multiplied the 3n by 4 but left the 2. Multiply the 2 by 4 too. Go to M3.8.4.
   - **If you chose B:** You added 4 to each term. The number outside multiplies, it does not add. Go to M3.8.4.
   - **If you chose D:** You multiplied only the 2. The 4 multiplies the 3n as well. Go to M3.8.4.

3. **4(2x + 3)**

   The highest common factor of 8 and 12 is 4. 8x ÷ 4 = 2x and 12 ÷ 4 = 3. So 8x + 12 = 4(2x + 3).

   - **If you chose A:** This equals 8x + 12, but 4x + 6 can still be divided by 2. Take out the highest common factor, which is 4. Go to M3.8.4.
   - **If you chose C:** You divided 8x by 4 but not 12. Divide every term by the common factor. Go to M3.8.4.
   - **If you chose D:** 8 does not divide 12 exactly. The common factor must divide both terms. Go to M3.1.3.

4. **4(x + 3)**

   One row is x + 3. There are 4 equal rows, so the picture is 4(x + 3).

   - **If you chose B:** This is 4 bars but only 3 squares. Every row has 3 squares, so there are 4 × 3 of them. Go to M3.8.4.
   - **If you chose C:** This has only one bar of x. The picture has 4 bars, one in each row. Go to M3.8.4.
   - **If you chose D:** The 4 rows multiply, they are not added. One row is x + 3, and there are 4 rows. Go to M3.8.4.

5. **The values are 15 and 7, so she is wrong.**

   3(x + 4) with x = 1 is 3 × 5 = 15. But 3x + 4 with x = 1 is 3 + 4 = 7. The values differ, so her answer is wrong. The right answer is 3x + 12.

   - **If you chose A:** The value of 3(x + 4) at x = 1 is 3 × 5 = 15, not 7. Work out the left side with the bracket first. Go to M3.8.2.
   - **If you chose B:** The value of 3x + 4 at x = 1 is 3 + 4 = 7, not 15. She forgot to multiply the 4 by 3. Go to M3.8.4.

6. **5x + 3**

   The x terms: 3x + 2x = 5x. The plain numbers: 4 − 1 = 3. So the answer is 5x + 3.

   - **If you chose A:** You added the 1 instead of taking it away. The sign in front of the 1 is minus. Go to M3.8.3.
   - **If you chose B:** You added 3x + 2x + 1 and dropped the 4. Collect the plain numbers on their own: 4 − 1 = 3. Go to M3.8.3.
   - **If you chose C:** You joined the plain numbers to the x terms. A plain number is not like a term with x. Go to M3.8.3.

7. **20**

   4(x + 5) = 4x + 4 × 5 = 4x + 20. The plain number is 20.

8. **5**

   The highest common factor of 10 and 15 is 5. So 10x + 15 = 5(2x + 3).

## M3.8.5. Solving equations in one step

1. **6**

   Take 3 from both sides. The bag balances 9 − 3 = 6 stones. Check: 6 + 3 = 9.

   - **If you chose A:** You added 3 and 9. The 3 is added to x, so take 3 from both sides. Go to M3.8.5.
   - **If you chose C:** That is the 3 loose stones on the left. The number x is what the bag holds. Go to M3.8.5.

2. **6**

   Divide both sides by 3. x = 18 ÷ 3 = 6. Check: 3 × 6 = 18.

   - **If you chose A:** You took 3 away from 18. But 3x means 3 times x. The opposite of times is divide. Go to M3.8.5.
   - **If you chose B:** You added 3 to 18. But 3x means 3 times x. The opposite of times is divide. Go to M3.8.5.
   - **If you chose C:** You multiplied 3 by 18. The 3 is already multiplying x, so divide both sides by 3. Go to M3.8.5.

3. **14**

   Add 5 to both sides. x = 9 + 5 = 14. Check: 14 − 5 = 9.

   - **If you chose A:** You took 5 from 9. But 5 was already taken away from x. The opposite is to add 5. Go to M3.8.5.
   - **If you chose B:** That is the answer to x − 5, not the number x. Add 5 to both sides to find x. Go to M3.8.5.

4. **12**

   Multiply both sides by 4. x = 3 × 4 = 12. Check: 12 ÷ 4 = 3.

   - **If you chose A:** You added 4 and 3. The x is divided by 4, so the opposite is to multiply by 4. Go to M3.8.5.
   - **If you chose C:** You divided 3 by 4. But x was divided by 4, so multiply both sides by 4. Go to M3.8.5.

5. **Take the same number of stones from both sides.**

   Equal things stay equal when you take the same from each. This is the one rule of solving equations.

   - **If you chose A:** Then the left side is lighter and the balance tips. Take the same from the other side too. Go to M3.8.5.
   - **If you chose B:** Then the left side is lighter and the balance tips. Whatever you do on one side, do on the other. Go to M3.8.5.
   - **If you chose C:** This makes the sides further apart. Do the same to both sides. Go to M3.8.5.

6. **23 + 8 = 31, not 15. His answer is wrong.**

   The correct answer takes 8 from 15: x = 15 − 8 = 7. Check: 7 + 8 = 15. Tariq added the 8 instead.

   - **If you chose B:** The equation says x + 8, not x − 8. Put 23 into x + 8. Go to M3.8.5.
   - **If you chose C:** The right side of the equation is 15. A true answer must give 15, not 31. Go to M3.8.5.

7. **2x + 6**

   2 × x = 2x and 2 × 3 = 6. So 2(x + 3) = 2x + 6.

   - **If you chose A:** You multiplied only the first term. The 2 multiplies the 3 as well. Go to M3.8.4.
   - **If you chose C:** You multiplied only the second term. The 2 multiplies the x as well. Go to M3.8.4.

8. **x = 11**

   Take 9 from both sides. x = 20 − 9 = 11. Check: 11 + 9 = 20.

9. **x = 7**

   Divide both sides by 6. x = 42 ÷ 6 = 7. Check: 6 × 7 = 42.

## M3.8.6. Solving equations in two steps

1. **5**

   Take 4 from both sides: 2x = 10. Divide by 2: x = 5. Check: 2 × 5 + 4 = 14.

   - **If you chose A:** You added 4 to 14. But the 4 is added to the 2x, so take 4 away first. Go to M3.8.6.
   - **If you chose B:** You divided 14 by 2 and took away 4. The 4 is not divided by itself. Take 4 from 14 first, then divide by 2. Go to M3.8.6.

2. **5**

   Add 3 to both sides: 4x = 20. Divide by 4: x = 5. Check: 4 × 5 − 3 = 17.

   - **If you chose A:** You took 3 from 17. But 3 was already taken away from 4x, so add 3 to both sides. Go to M3.8.6.
   - **If you chose C:** You added 3, which is right, but you stopped. This is 4x = 20. Divide both sides by 4. Go to M3.8.6.

3. **4**

   Divide both sides by 2: x + 3 = 7. Take 3 from both sides: x = 4. Check: 2 × (4 + 3) = 14.

   - **If you chose B:** This is the value of 2x, not x. You still need to divide both sides by 2. Go to M3.8.6.
   - **If you chose C:** You took 3 from 14. But the 2 multiplies the whole bracket. Divide 14 by 2 first, or expand the bracket. Go to M3.8.6.

4. **4**

   Take 3 bags from both sides: 2x = 8. Divide by 2: x = 4. Check: 5 × 4 = 20 and 3 × 4 + 8 = 20.

   - **If you chose A:** After you take 3 bags from each side, 2 bags balance 8 stones. That is 2x = 8. Now divide by 2. Go to M3.8.6.
   - **If you chose B:** You added 5 and 3, then divided 8 by that. The bags are taken from both sides, 5 − 3 = 2. Go to M3.8.6.

5. **Take the loose number away from both sides.**

   We undo the steps backwards. The loose number was added last, so we take it away first. Then we divide.

   - **If you chose A:** Then the sides are no longer equal. Whatever you do, do it to every term on both sides. Go to M3.8.6.
   - **If you chose B:** The loose number is already added to the bags. To undo adding, take it away. Go to M3.8.6.
   - **If you chose C:** Multiplying makes the numbers bigger and does not free x. Undo the adding first. Go to M3.8.6.

6. **2 × 4 + 1 = 9, not 11. Her answer is wrong.**

   The right answer is x = 5, because 2 × 5 + 1 = 11. Leila forgot to divide 10 by 2.

   - **If you chose A:** The right side of the equation is 11. A true answer must make the left side 11, not 9. Go to M3.8.6.
   - **If you chose C:** 2x means 2 times x. Put 4 in place of x, then multiply: 2 × 4. Go to M3.8.2.

7. **16**

   Add 6 to both sides. x = 10 + 6 = 16. Check: 16 − 6 = 10.

   - **If you chose B:** You took 6 from 10. But 6 was already taken away from x, so add 6 to both sides. Go to M3.8.5.
   - **If you chose C:** You multiplied 6 by 10. The opposite of taking away is adding. Go to M3.8.5.

8. **x = 5**

   Take 5 from both sides: 3x = 15. Divide by 3: x = 5. Check: 3 × 5 + 5 = 20.

9. **x = 6**

   Divide both sides by 3: x + 2 = 8. Take 2 from both sides: x = 6. Check: 3 × (6 + 2) = 24.

## M3.8.7. Writing your own equations

1. **n + 4 = 15, so n = 11**

   Joseph gets 4 more, so n + 4 = 15. Take 4 from both sides: n = 15 − 4 = 11. Check: 11 + 4 = 15.

   - **If you chose A:** A minus sign means he gave 4 away. Joseph gets 4 more, so we add. Go to M3.8.7.
   - **If you chose C:** 4n means 4 times n. Joseph does not have 4 boxes. He gets 4 more beans. Go to M3.8.1.

2. **2n − 3 = 11, so n = 7**

   Add 3 to both sides: 2n = 14. Divide by 2: n = 7. Check: 2 × 7 − 3 = 11.

   - **If you chose B:** You missed the doubling. The number is doubled first, so we write 2n. Go to M3.8.7.
   - **If you chose C:** She takes 3 away. A plus sign would add 3. Go to M3.8.7.

3. **4 m**

   The equation is 4x + 3 = 19. Take 3 from both sides: 4x = 16. Divide by 4: x = 4. Check: 4 × 4 + 3 = 19.

   - **If you chose A:** You took 3 from 19, which is right, but you stopped. 16 m is all 4 pieces. Divide by 4. Go to M3.8.7.
   - **If you chose B:** You added 3 to 19 and divided by 4. The 3 m is part of the rope, so take it away from 19. Go to M3.8.7.

4. **5 × 7 + 2 = 37, not 32. She is wrong.**

   The right answer is x = 6, because 5 × 6 + 2 = 32. The 7 does not fit, so Lucía made a slip.

   - **If you chose A:** The right side is 32. A true answer must make the left side 32, not 37. Go to M3.8.7.
   - **If you chose C:** 5x means 5 times x. Put 7 in place of x, then multiply: 5 × 7. Go to M3.8.2.

5. **2l + 12 = 30**

   P = 2l + 2w with w = 6 gives 2l + 2 × 6 = 30, so 2l + 12 = 30. Then 2l = 18 and l = 9.

   - **If you chose A:** This adds only one length and one width. The perimeter goes round all four sides. Go to M3.8.2.
   - **If you chose B:** You doubled the length but not the width. There are two widths, so 2 × 6. Go to M3.8.2.
   - **If you chose C:** This multiplies the sides, which is the area. The perimeter adds the four sides. Go to M3.8.2.

6. **5**

   Take 4 from both sides: 3x = 15. Divide by 3: x = 5. Check: 3 × 5 + 4 = 19.

   - **If you chose B:** You took 4 from 19, which is right, but you stopped. This is 3x = 15. Divide by 3. Go to M3.8.6.
   - **If you chose C:** You added 4 to 19 and stopped. The 4 is added to 3x, so take it away. Go to M3.8.6.

7. **5**

   The equation is 3n + 4 = 19. Take 4 from both sides: 3n = 15. Divide by 3: n = 5. Check: 3 × 5 + 4 = 19.

8. **19 beans**

   The equation is n − 7 = 12. Add 7 to both sides: n = 19. Check: 19 − 7 = 12.

## M3.8.8. Sequences and the nth term

1. **3n + 1**

   The terms go up by 3, so start with 3n. The terms of 3n are 3, 6, 9, 12. Each term is 1 more. So the nth term is 3n + 1.

   - **If you chose A:** Test n = 2. Then n + 3 is 5, but the second term is 7. The difference 3 goes in front of n. Go to M3.8.8.
   - **If you chose B:** The terms of 4n are 4, 8, 12, 16. This goes up by 4, but the sequence goes up by 3. Go to M3.8.8.
   - **If you chose C:** The terms of 3n are 3, 6, 9, 12. Each term of the sequence is 1 more than that. Go to M3.8.8.

2. **152**

   Put n = 50 into 3n + 2. 3 × 50 = 150, and 150 + 2 = 152.

   - **If you chose B:** You put 3 and 50 side by side as 350. But 3n means 3 × n, so 3 × 50 = 150. Go to M3.8.2.
   - **If you chose C:** You added 3 + 50 + 2. A number next to a letter means multiply: 3 × 50. Go to M3.8.2.

3. **Yes. 3n + 2 = 62 gives n = 20, a whole number.**

   Solve 3n + 2 = 62. Take 2 from both sides: 3n = 60. Divide by 3: n = 20. It is a whole number, so 62 is the 20th term.

   - **If you chose A:** The list shows only the first four terms. The three dots mean it goes on for ever, so use the nth term. Go to M3.8.8.
   - **If you chose C:** The rule is 3n + 2, not 3n. Take 2 from 62 first. Then 60 ÷ 3 = 20. Go to M3.8.6.

4. **4n + 3**

   The terms go up by 4, so start with 4n: 4, 8, 12, 16. Each term is 3 more. So the nth term is 4n + 3.

   - **If you chose A:** The number added to 4n is 7 − 4 = 3, not 7. The terms of 4n are 4, 8, 12, 16, and each term is 3 more. Go to M3.8.8.
   - **If you chose B:** Test n = 2. Then n + 4 is 6, but the second term is 11. The difference 4 goes in front of n. Go to M3.8.8.

5. **3**

   8 − 5 = 3, and 11 − 8 = 3, and 14 − 11 = 3. The terms go up by 3 each time.

   - **If you chose B:** That is the number added in 3n + 2. The difference is the step between terms. Go to M3.8.8.
   - **If you chose C:** That is the first term. The difference is how much the terms go up by each time. Go to M3.8.8.

6. **n + 3 gives 5 for n = 2, but the second term is 8. He is wrong.**

   The right rule is 3n + 2, which gives 3 × 2 + 2 = 8 for n = 2. So n + 3 is wrong.

   - **If you chose A:** The second term of the sequence is 8, not 5. A correct nth term must give 8. Go to M3.8.8.
   - **If you chose C:** Put in n = 2 again. 2 + 3 is 5, not 8. Go to M3.8.2.

7. **6**

   Take 7 from both sides: 2x = 12. Divide by 2: x = 6. Check: 2 × 6 + 7 = 19.

   - **If you chose A:** You took 7 from 19, which is right, but you stopped. This is 2x = 12. Divide by 2. Go to M3.8.6.
   - **If you chose B:** You added 7 to 6, or took 6 from 19. Take 7 from both sides first, then divide by 2. Go to M3.8.6.

8. **83**

   Put n = 20 into 4n + 3. 4 × 20 = 80, and 80 + 3 = 83.

9. **5**

   11 − 6 = 5, and 16 − 11 = 5, and 21 − 16 = 5. The terms go up by 5 each time.

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# About this book

For learners who can work out 3 + 4 × 2 in the right order. In many schools this fits children of about 11 to 13, and older learners who never met letters in mathematics can start here too. Every new step is shown as a picture before it is written with letters. Each chapter takes one or two short sessions. A learner can work alone: every exercise has its answer and the reason for it. Each chapter also has a short page for the adult, with three questions to ask.

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